

# Holonomy groups of Lorentz-Kähler manifolds

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The holonomy group of a pseudo-Riemannian manifold  $(M, g)$  is an important invariant that gives rich information about the geometry of  $(M, g)$ . This motivates the classification problem for holonomy groups of pseudo-Riemannian manifolds. The problem is solved only for connected holonomy groups of Riemannian and Lorentzian manifolds [1]. We obtain the classification of the connected holonomy groups for pseudo-Kählerian manifolds of complex index one [2]. In particular, we construct metrics with each possible connected holonomy group by giving their pseudo-Kählerian potential.

## References

- [1] Galaev, A., Leistner, Th. Recent developments in pseudo-Riemannian holonomy theory. Handbook of pseudo-Riemannian geometry and supersymmetry, 581–627, IRMA Lect. Math. Theor. Phys., 16, Eur. Math. Soc., Zürich, 2010.
- [2] Galaev, A.S. Holonomy classification of Lorentz-Kähler manifolds. arXiv:1606.07701.

Holonomy groups of Lorentzian manifolds (= space-times). Let  $H$  be the holonomy group of a space-time of dimension  $n + 2$  that is not locally a product. Then, either  $H$  is the full Lorentz group  $SO(1, n + 1)$  [Berger 1955] or  $H \cong (R \times SO(n)) \ltimes R^n$  = stabiliser of a null line,  $G := \text{pr}SO(n)(H)$  is a Riemannian holonomy group [TL, J. Differential Geom., 1977]  $H = G \ltimes R^n$  or  $H = (R \times G) \ltimes R^n$ , or  $(L \times S) \ltimes R^k$ , where  $S$  is the semisimple part of  $G$  and  $L \cong R \times Z(G)$  or  $L \cong R \times K \ltimes Z(G)$  [Beard-Bergery & Ikemakhen, Proc. Symp. Pure Math. References. Lorentzian manifolds with transitive conformal group. Dmitri alekseevsky. A.A.Kharkevich Institute for Information Transmission Problems B.Karetnuj per., 19, 127051, Moscow, Russia e-mail : dalekseevsky@iitp.ru. Abstract. 3. Conformally homogeneous Lorentz manifolds of type A. 3.1. Conformally homogeneous manifolds associated with graded subalgebra of  $\mathfrak{so}(n+1, 1)$ . Let  $\mathfrak{g} = \mathfrak{g}^{-1} + \mathfrak{g}_0 + \mathfrak{g}_1 = \mathfrak{V} + \mathfrak{g}_0 + \mathfrak{g}_1$  be a graded subalgebra of the graded Lie algebra  $\mathfrak{so}(n+1, 1) = \mathfrak{V} + \text{co}(\mathfrak{V}) + \mathfrak{V}$ . [G] Galaev A.S., Conformally homogeneous Lorentzian manifolds with special holonomy groups, Mat. Sb., v 204, n 9, 29-50, 2013. [P-R] Penrose R., Rindler W., Spinors and Space-Time, v 2, 1986. Kähler Immersions of Kähler Manifolds into Complex Space Forms. Vol. 23, Issue. , p. 29. CrossRef. We prove that if the normal holonomy group acts irreducibly on the normal space then it is linear isomorphic to the holonomy group of an irreducible Hermitian symmetric space. In particular, it is a compact group and the complex structure  $J$  belongs to its Lie algebra. We prove that the normal holonomy group acts irreducibly if the submanifold is full (that is, it is not contained in a totally geodesic proper Kähler submanifold) and the second fundamental form at some point has no kernel. For example, a Kähler-Einstein submanifold of  $\mathbb{C}P^n$  has this property. We define a new invariant  $\mu$  of a Kähler submanifold of a complex space form. HOLONOMY THEORY AND 4-DIMENSIONAL LORENTZ MANIFOLDS by G.S. Hall advertisement. UNIVERSITATIS IAGELLONICAE ACTA MATHEMATICA, FASCICULUS XLI 2003 HOLONOMY THEORY AND 4-DIMENSIONAL LORENTZ MANIFOLDS by G.S. Hall Abstract. This lecture describes the holonomy group for a 4-dimensional Hausdorff, connected and simply connected manifold admitting a Lorentz metric and shows, briefly, some applications to Einstein's space-time of general relativity. 1. Holonomy Theory on 4-Dimensional Lorentz Manifolds. In the present paper, holonomy algebras of Lorentz-Kähler manifolds are classified. A simple construction of a metric for each holonomy algebra is given. Complex Walker coordinates are introduced and described using the potential. Complex pp-waves are characterized in terms of the curvature, holonomy, and the potential. Classification of Lorentz-Kähler symmetric spaces is reviewed. This is a preview of subscription content, access via your institution. Access options. Baum, H.: Holonomy Groups of Lorentzian Manifolds: A Status Report. Global Differential Geometry. Springer Proceedings in Mathematics & Statistics, vol. 17.