

Tunneling in nearly integrable systems with a non-hermitian perturbation

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We study the tunneling effect in nearly integrable systems with a non-hermitian perturbation. In the integrable system such as a particle moving in the one-dimensional double well potential, the energy splitting ΔE caused by quantum tunneling is evaluated as

$$\Delta E \underset{\hbar \rightarrow 0}{\sim} e^{-S/\hbar}, \quad (1)$$

by the semiclassical (WKB) approximation, where the action S is determined by the classical quantity [1, 2]. On the other hand, in the systems under the periodic perturbation, the corresponding classical system becomes non-integrable. If one plots the energy splitting as a function of $1/\hbar$, it exhibits persistent enhancement from the prediction (1) accompanying spikes.

The spike can be interpreted as energetic resonance with excited states by photon absorption in the language of quantum dynamics, but it may be reinterpreted by the language of classical dynamics. The theory of resonance-assisted tunneling (RAT) have discussed the relation between the appearance of the spikes and the classical non-linear resonances, and then it claimed that the classical non-linear resonances create a bunch of spikes, which brings the enhancement of tunneling probability [3, 4].

The appearance of the spikes in the energy splitting has been considered as the origin of the enhancement of tunneling probability, but to make clear this issue, we introduce a weak non-hermitian perturbation which pushes the resonant states to the complex domain. By applying this perturbation, we found the spikes and the persistent enhancement have the different origin, and it was unveiled that the staircase-like structure is hidden in the energy splitting curve as a function of $1/\hbar$ [5].

References

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The enhancement of tunneling probability in the nearly integrable system is closely examined, focusing on tunneling splittings plotted as a function of the inverse of the Planck's constant. On the basis of the analysis using the absorber which efficiently suppresses the coupling, creating spikes in the plot, we found that the splitting curve should be viewed as the staircase-shaped skeleton accompanied by spikes. We further introduce renormalized integrable Hamiltonians and explore the origin of such a staircase structure by investigating the nature of eigenfunctions closely. We further introduce renormalized integrable Hamiltonians and explore the origin of such a staircase structure by investigating the nature of eigenfunctions closely. We further introduce renormalized integrable Hamiltonians and explore the origin of such a staircase structure by investigating the nature of eigenfunctions closely.

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3Dynamical Tunneling Theory and Experiment Univ-tours.fr Tunneling is described by a transmission coefficient which gives the ratio of the current density emerging from a barrier divided by the current density incident on a barrier. In order to produce a non-vanishing current density, the wave function must have a position dependent phase. Otherwise, the phase of $\hat{\psi}(x, t)$ will be the same as the phase of $\hat{\psi}(x, t)$ and therefore $\hat{\psi}^* \hat{\psi}$ will be real. The current density for an electron in a stationary state of the form $\hat{\psi}(x, t) = \hat{\psi}(x) \exp(i\omega t)$ is zero since the phase dependence has no spatial dependence. Quantum Tunneling in the Real World. There are a wide variety of real world phenomena which can be pictured in terms of tunneling processes. The Floquet Hamiltonian H_{eff} is Hermitian, so the Floquet eigenvalues ϵ_{\pm} are real and two Floquet eigenstates $|\hat{\psi}_{\pm}\rangle$ and $|\hat{\psi}_{\pm}^*\rangle$ belonging to different eigen-values are orthogonal. Additionally, the Floquet Hamiltonian commutes with the parity operator denoted by its action on the momentum eigenket $|\hat{\psi}_{\pm}\rangle = |\hat{\psi}_{\pm}\rangle$. Therefore the two operators can be diagonalized simultaneously and all Floquet eigenstates have definite parity: $\hat{P}|\hat{\psi}_{\pm}\rangle = \pm|\hat{\psi}_{\pm}\rangle$. The perturbed eigenstates and eigenvalues at $\epsilon = \epsilon_0$. The details of the perturbation analysis are given in Appendix A. The results may be summarized as follows. Predictions of perturbation theory for a number of other avoided crossings in this system (as well as those in systems with different values of ω), finding similar agreement.

8.1.1 Tunneling in integrable systems Since the early days of quantum mechanics, tunneling has been recognized as one of the hall-marks of the wave character of microscopic physics. The approach presented in the previous section can be generalized to multidimensional, even non-separable systems, as long as their classical dynamics is still integrable [8]. It breaks down, however, as soon as a non-integrable perturbation is added to the system, e.g. if the one-dimensional double-well potential is exposed to a driving that is periodic in time (with period T).