

Chapter 8

Teaching and Learning in Secondary Mathematics Classroom

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Abstract

This article attempts to explain general view of Korean secondary mathematics classroom instruction, and as samples five mathematics lessons are introduced. One of these five lessons is videotaped.

1. Introduction

Secondary school mathematics education in Korea is very different from elementary level not only in the depth of the content but also in attitudes of students toward mathematics education. Most secondary school students concern more about their grades since they affect their future opportunities of higher education directly. Recently, secondary mathematics education classrooms have been dramatically changed in instructional approaches. Many teachers are focusing more on student-oriented, exploration-oriented activities in their lessons. It is more prevalent after the 7th curriculum, but even before that many mathematics teachers attempted to improve their teaching methods with a belief that student-centered instruction is the best way to achieve their goals of attaining students' understanding of mathematical concepts.

This article will introduce mathematics lessons involving student-centered, exploration-oriented activities. One thing readers should keep in mind is that teachers do not use one method during a semester, and they utilize different teaching methods depending on content areas. They could use traditional methods in one lesson, and use manipulative materials or exploration activities in other lessons.

II. General View of Secondary Mathematics Classroom

1. Characteristics of mathematics lessons

(1) General Characteristics

Generally speaking, there is not much difference between elementary and secondary classrooms. In secondary level, students have different teachers depending on the subjects, and the content of each subject is more profound. In elementary school, one period lasts 40 minutes. It is 45 and 50 minutes in middle school and high school, respectively. Recess time is 10 minutes at all levels.

(2) Characteristics of Mathematics Lessons

1) Teachers use textbooks as a main source of instruction, but sometimes they use activity sheets or problems set as supplementary sources. In this level, students do not use workbooks that had been used in elementary level.

2) The contents of mathematics textbooks are more abstract. New symbols, letters, generalizations, proof, and reduction to absurdity are introduced.

3) The contents of mathematics become complex as the grades go up. More students lose their interest in mathematics and it is getting worse in high school. Many students complain that mathematics is too difficult to understand.

4) College bound students recognize mathematics as one of the main subjects to prepare for college entrance examination and study mathematics everyday at school, but mathematics is not an important subject for students who intend to find a job after graduation from high school.

2. Methods of mathematics lessons

Students are involved in many activities such as listening, asking questions, discussions, observations, experiments in the mathematics classroom. Recently more teachers are focusing on student-centered activities, and spend more time on visual resources such as TV or videotaped educational sources. Many teachers also open their classroom teaching to their colleagues to improve their instructional skills.

Many different types of instruction (e.g. individual instruction, differentiated instruction according to students' mathematical ability, computer assisted instruction, instruction involving games, instruction with manipulative, instruction involving readings and instruction with mind map etc.) can be implemented according to the following questions. What kinds of content should be introduced? What kind of instructional approaches should be used to facilitate students' involvement in the lessons? What should be done to overcome the differences among students in their mathematical abilities?

3. Assessment in mathematics learning

Only few years ago, most of assessment was only paper-and pencil tests. In this case, many teachers were discouraged since many students could perform well without paying attention to the lessons. On the one hand, they mastered the topics in their private lessons. On the other hand, some students pay attention to the lessons with positive attitudes, but there was no way to offer any credits if they failed on the tests. During the last few years, dramatic changes in assessment have been reflecting portfolio, reports and attitudes in assessment. In mathematics, assessment includes student participation, attitudes, individual and group reports.

4. Overview of mathematics teachers

Previously, teachers' body was based on administration structure, but nowadays many schools organize teachers' body according to the subject area. For example, traditionally administration units were departments only in Physical Education and Sciences. Recently this trend is extending to Mathematics, Languages, Social Studies and Art department, even though there are some variances among schools. In this system, teachers have regular meetings and discuss current issues in instruction and assessment methods in schools. It is also not unusual that teachers participate in teacher organization of school districts or national organization to share ideas among teachers and improve their instructional methods.

5. Extracurricular activities

Most students are taking private lessons in music, art, physical education, language, science, and mathematics. Specifically, many students take mathematics lessons at the private institutions or from private tutors depending on the financial background of the parents.

III. Examples of Middle School Mathematics Lessons

What would it be like Korean mathematics lessons? I would like to introduce a few video scenes containing mathematics lessons in Korea. It will give some pictures of Korean mathematics classroom.

1. Mr. Kim-1 focusing on mathematical attitudes

Grade and Topic : 8th grade, linear function

(1) Characteristics of the Lesson

Mr. Kim reorganized the curriculum, and made it possible for the teacher and his students to do their parts. Most of the students like to study with new materials compared with textbooks. There are three stages in his lessons, 'Opening' focusing on understanding basic concepts and terminologies, 'Makes It My Own' emphasizing assimilation of the contents, and 'Wrapping It Up' focusing on connecting various ideas with the lessons.

In reality, it is not easy to reorganize the curriculum in every lesson. Therefore he rearranges the chapters into three components. During the lesson he provides work sheets that were designed to relate key ideas and to facilitate practicing those ideas. He also frequently adopted cooperative learning and self-paced instructions.

(2) Characteristics of Teaching and Learning Methods

1) Differentiated instruction

In this lesson, both teacher and the students feel comfortable since the instructional methods fit each student's mathematical ability. Every three months, students are assigned to new groups. Each group consists of students who reveal different levels in mathematics but they tend to possess similar personal characteristics. Sometimes a few students who need special attentions are assigned to a special group. These arrangements become a base for students' active involvements in the lessons. Also Mr. Kim presents students with 'stickers' as awards and it is also included in class credits as a part of performance assessments.

In planning the lessons the ratios of the contents according to the levels are 2(high) to 5(average) to 3(low). Teacher prepares additional problems not only for students who understood the contents to study profoundly but also for the low learners to practice the topics.

Students' responses are very positive. Since this instructional method is implemented in a classroom, there are no negative responses from the students. Mr. Kim was proud of positive effects of his method because group activities with students of various levels of mathematics would cultivate students' attitude of cooperation and also develop communication skills through solving problems with group members. He also mentioned that this method is effective in nurturing positive attitudes toward mathematics, and helpful in group performance assessment because teacher does not need to explain basic concepts. Students solved the problems through group discussions. There are many positive aspects in this method, yet many difficulties follow

in implementation. First of all, it is challenging to prepare the lessons appropriate for students' characteristics.

2) Lessons with natural questions

Mathematical knowledge acquired through communications between a teacher and students would last longer and be easy to apply. Students express their own ideas and listen to others' opinions. In 45 minutes, Mr. Kim only uses 15 minutes to ask questions and listen to the students, and rest of the time, all the students lead the discussion. Students recognized that communication of mathematics and mathematics lesson is meaningful process.

Teachers' questions are used as a stimulus to elicit related mathematical concepts students already knew. Naturally elicited knowledge could be developed concretely compared with artificial methods. In this viewpoint, teachers should ask appropriate questions. This is about Mr. Kim asking questions regarding functions students already learned in the previous year to induce motivation and interests. In explanations teacher used familiar examples that are linked to mathematical concepts and it allows students to approach the contents easily. Teachers' questions are helpful in leading students' mathematical thinking and working as an aid.

3) Cooperative learning

The reason for implementing cooperative learning is to render opportunities for students to experience society in which individuals communicate and share their own opinions to live together. This method of teaching made it possible to understand the teacher's educational philosophy focusing on encouragement, cooperation, and communications through expressing students' opinions. This aspect was more emphasized on the lessons using performance assessment, which would develop students' various abilities.

4) Self-paced instruction

This method was utilized to allow for the teacher to teach both advanced students and slow learners. They could learn a topic according to their paces. In this teaching approach, we could see the teacher's intention of accepting differences among students in abilities. It is emphasized in the classrooms as classified lessons according to students' abilities through reconstructed and motivation-elicited lessons.

(3) Characteristics of Assessment

1) Performance assessment

Mr. Kim said, "In implementing performance assessments, I talk to myself that teaching is assessment, and assessment is teaching." He attempted to assess students' performance from the beginning stage of students' learning to the stage where he knows how students understood the topic. He viewed assessment as a resource of understanding students' learning processes rather than as an evaluation at the end of the instruction. Kr. Kim also said, "The process of understanding mathematics is more important than the results. I don't think people will be evaluated according to the amount of knowledge they possess in the future." He intends to assess students' attitudes, motivations, and willingness.

Accordingly, Mr. Kim is also considerate in constructing the contents of instruction. In introduction, it is emphasized that mathematics is something we need. At the beginning it was designed for students to know the importance of the topic. In the middle part of the lesson, it is designed to study each topic. At the end an attempt was made to connect all the processes and concepts. In addition, in order to include performance assessment in the instruction, he used cooperative learning, experimental reports, and qualitative assessment methods involving their attitudes and contributions in the classroom.

2) Grading on motivation and attitude

The teacher intended to evaluate students as a whole using various assessment methods. Accordingly, he evaluated not only the results of students' achievement but also their motivations, eagerness, and attitudes in learning mathematics. He said it was prerequisite to understand students to implement this assessment method. He did his best to evaluate students' achievements holistically. The following is his explanation expressing his intention.

First of all, I tried to understand students through observation. I encourage and praise students based on their attitudes, taking notes and the traces of solving problems. Also I do not skip short oral tests to each student in every class. Therefore students could understand the topics thoroughly in each lesson. In addition, we play soccer and eat lunch together to establish intimate rapport.

(4) Stages in Lesson

Mr. Kim practiced various instructional activities with introduction - development - conclusion stages. He used familiar words for these stages; opening, makes it my own, wrapping it up. And he utilized various activities to lead the students to the goals of the lessons. For example, in introduction, he used explorations, investigations, and experiments to elicit motivations and interests. At the end of each lesson, students could examine themselves with handouts. And at the end of sub-chapter, some amount of time was allocated for students to summarize what they had learned. Students were cooperative to Mr. Kim's instructional methods.

1. Introduction	Development	Consolidation and Assessment
	<ul style="list-style-type: none"> • Opening Mind 1 • Organizing • Opening Mind 2 	<ul style="list-style-type: none"> • Conclusion • Formative Evaluation

2. Ms. Park 1's lesson eliciting motivation through manipulative activities

Grade and Topic: 7-2, Measurement in Geometry

Ms. Park-1 implemented " motivation-elicited lessons" using concrete manipulative and the history of mathematics. As a part of her lesson, group cooperative activities and discovery lesson were also found.

(1) Characteristics of Lesson

1) Introduction of history of mathematics

Ms. Park1 said that she explained the historical background of the chapter before studying a new chapter, and shared the difficult experience of mathematicians. She also made activity sheets that would help students to understand mathematical symbols and the history of mathematics. She began a new chapter with the historical background in order to elicit motivations of the students. She thought that it is more important to introduce the history of mathematics at the beginning of a new chapter rather than sub-chapter.

She introduced the life of mathematicians and distributed a summary of their contributions in mathematics. By doing it, students felt like listening to interesting stories, and they could reveal positive attitudes toward mathematics lessons.

2) Discovery lesson by teachers' guided questions

Most teachers in Korea lead their classes based on expository lessons. Ms. Park1 said that one-way explanation would reduce students' interests, and it is important for teachers to ask questions to stimulate motivation.

When the teacher's explanation was necessary, I did it. But it was easy for students to lose interests; therefore I prepared the lessons with many resources. The most frequent use of methods was asking questions during the class. This communications usually become proper motivation before studying main topics. Also this method not only checks individual student and resolves the problems but also it becomes an opportunity to let students who are the same level know themselves through this process. These questions render students time to think about their thinking process and check and confirm their achievement levels.

3) Lesson focusing on students' motivation

When asked what would she do in case she thought the current curriculum and content were not appropriate for students' levels, Ms. Park1 said that she would reconstruct the content of the textbooks to fit students' levels. In this case, she focused on the core content, and introduced the topics with historical background, also prepared activity sheets that were designed to group activities with manipulative.

(2) Characteristics of Teaching and Learning Methods.

1) Lesson focusing on students' activities

Ms Park1 has constructed her own instructional activity sheets that were designed to fit the levels of the students after completing examination of the textbooks and teachers' manual. She also planned her lessons emphasizing small group activities with interesting contexts and students-centered instructions. Most of the activities were to use manipulative.

Students paid more attention to activities- oriented instructions even though those were not fancy materials at all. I reconstructed these guidelines based on teachers' manual and textbooks of foreign countries.

Ms. Park1 prepared worksheets, dot papers and colored papers for the students. And in the lesson, she has assigned most of the time for students' activities except teachers' explanations and questions.

2) Instruction emphasizing group cooperation

The instruction emphasizing group cooperation can let students realize that everyone plays his or her own role, encourage them to arrive at agreement based on questions and answers within the group, and help them accept others' strengths and weaknesses. Cooperative instruction also is useful for students, in particular who are rather stubborn persisting on their own ideas, to listen carefully to others' ideas and to examine different methods to approach a given problem. It is easy for students to forget knowledge accumulated by receiving passively what the teacher says. But it is relatively easy to remember and thus to use knowledge acquired by questions, explorations, and discussions on the part of the students. Ms. Park1 knows well these benefits of group cooperation in the mathematics classroom, and in fact often employs many activities based on group cooperation.

3) Permissive classroom atmosphere

Ms. Park1 gives her students a lot of opportunities to discover important mathematical concepts or principles for themselves and provides them with positive feedback. The more the teacher applauded, the more interested students become in the lesson. At the end of the instruction, the teacher offers individualized guidance as a part of her attempts to overcome individual differences among students. Partly because of the number of students in one classroom and time limit, she often finds it difficult to employ individualized teaching but she also admits its benefits for the sake of students' learning.

(3) Characteristics of Assessment

1) Performance assessment using the history of mathematics

Ms. Park1 often uses the problems in the history of mathematics to which students are urged to apply what they have learned rather than to recall it. Performance assessment plays an important role in her classroom in that students become familiar with the history of mathematics and furthermore make use of it. The history of mathematics makes students motivated to learn mathematics and realize how it is connected to real-life situations or applications.

2) Various assessment methods

Ms. Park1 applies different assessment methods to her lesson. She uses assessment both to monitor students' progress and to get a feedback for her instruction. When asked what kinds of assessment methods she uses, Ms. Park1 answered:

On the basis of the contents and characteristics of the unit, I use the "questions and answers" method by which I ask simple but critical questions on what students have learned and they need to answer immediately. I also use worksheets for assessment during the lesson. Basically I attempt to check whether or not students have learned what they are supposed to learn during the session. I sometimes use 'observation' method by which individual students are assessed in detail on how they solve a given fundamental problem. In particular, this method is used to ascertain whether students have understood the most essential part of the lesson. I also sometimes ask for self-assessment in a mathematics journal so as for students to look at and reflect on their own learning.

The teacher does not use just one assessment method. She uses rather traditional assessment methods, both the "questions and answers" method and the worksheets method, in order to see how much the instructional goals of the lesson have been achieved. Note that she also employs the observation method so as to meet with individual students' needs. The teacher consistently guides students to write on a mathematics journal by which she explores their interest in and motivation towards mathematics. The teacher also urges students to include their own assessment on mathematics learning. It usually takes 10 out of 45 minutes (about 20%) for her to assess students during the lesson. It should be emphasized that the results of assessment are employed to adjust her instruction.

(4) Overview of the Lesson

Introduction	Development	Consolidation and Evaluation
<ul style="list-style-type: none"> • Confirmation of what was learned before • Elicitation of students' motivation 	<ul style="list-style-type: none"> • Students' activities • Presentation of students' methods • Solution of the worksheets tailored to individual students' mathematical abilities 	<ul style="list-style-type: none"> • Teacher's explanation • Assessment by a worksheet

3. Ms. Park2's lesson focusing on meta-cognition with mind-map

Grade and Unit: 8-2, III. Similarity of figures

Mind-map makes it easier for students to know what is learned and to connect it with what was learned and what will be learned. Mind-map is a useful meta-cognitive strategy in that it facilitates thinking about thinking, or reflection on thinking.

(1) Overall Characteristics

The main characteristic is to use mind-map before and after the lesson so that students become familiar with the overall overview and connect related concepts or principles one another. Ms. Park2 explains mind-map:

The purpose of learning mathematics in school is not to make all students be mathematicians. Rather it is to help them regard mathematics as a useful tool in their everyday life, learn to think logically by mathematics, and communicate with others. In order to do that, students need to know which part of mathematics they are learning and how it is connected to others or real-life contexts. Mind-map is good for this.

(2) Characteristics of Instruction

1) Preparation

It usually takes approximately 2 hours for Ms. Park2 to prepare for one lesson of 45 minutes. She teaches three different classes with the same lesson plan. She spends almost all the extra time planning, editing, and typing worksheets for her lessons, and examining students' reports. Ms. Park2 prepares her lesson in the following order: First, she makes a mind-map on what is taught. Second, she determines a teaching sequence and represents it in the mind-map on the basis of what students have already learned, what they may be easy to overlook, and what they should be taught in a given unit. Third, she makes it sure to include the essential part of mathematics in the unit. Fourth, she poses some questions in advance on what students often make a mistake.

Ms. Park2 says that a mind-map prepared in this way is one of the most effective instructional devices in that it may combine both already-made worksheets and new ideas.

2) Concerns of students' emotional states

Ms. Park2 first approaches to her students not as a strict mathematics teacher but as a kind person. She attempts to observe what the students do and provide them with any help, listen

carefully to what they say, and remember who an individual student is. The teacher believes that students tend to be interested in mathematics, even though they do not like it at first, as they come to like their mathematics teacher. She also finds that students know many mathematical facts but do not know when and how to use them. So she tries not only to guide but also to think with them on that matter. In this way, Ms. Park² regards her relationship with the students not as a provider and receivers of information but as comrades of seeking for knowledge.

It is always my concern that every child has his or her strength and I have to give a lot of opportunities by which an individual child expresses his or her strength in the mathematics classroom. In my class, I ask a logical question to a student who likes logic. I also ask a question requiring space sense to a student who is good at that sense. In the same way, I ask a student who is excellent in linguistic ability to create a mathematics story related to the given lesson. In this way, I attempt to nurture students' self-confidence in mathematics on the basis of their own overall strengths.

3) Emphasis on mathematical connections through mind-map

Ms. Park² begins a lesson by presenting a mind-map and confirming both what was covered and what will be covered. In this way, she uses a mind-map for students to see mathematical connections across lessons. As meeting with different students every year, Ms. Park² has been attempting to apply various teaching methods, which are probably compatible with the characteristics of the students. Among many methods, Ms. Park² explains that a mind-map is an excellent tool with which students learn how to use mathematics in everyday life, how to think logically, and how to relate mathematical facts to one another.

4) Considering students' aptitude

Ms. Park² thinks that students are different not only by mathematics achievement but also by what they like and how much they do. Whenever she asks a question, therefore, the teacher considers individual students' aptitude or disposition.

5) Reconstructing the curriculum for understanding

As described above, instruction using a mind-map often leads to reconstruct the curriculum and textbooks. It is not solely the teacher but the students who draw a mind-map per unit, as

thinking of what and why they learn. The teacher explains how she reconstructs the textbooks:

First I spend much time analyzing what the curriculum intends to teach and what the textbooks facilitate to teach at a given grade. Because even cursory examination of 16 kinds of textbooks published under the current curriculum reveals different structures and methods, the reconstruction of textbooks tailored to my students is really important. In reconstructing textbooks, I consider the overall flow of thinking, affection aspects, and mathematical knowledge with skills. In particular, I pay attention to make the flow of thinking natural on the part of the students by connecting what was learned, what is learned, and what will be learned.

6) Instruction through communication

As a part of her attempts to increase students' understanding of concepts, Ms. Park2 emphasizes communication in the mathematics classroom. For instance, she asks students to present their own ideas, to comment on others' thinking, to come up with a new approach, and to arrive at an idea agreed upon through enough discussion. Ms. Park2 also thinks that it is important for students to realize the connection between mathematical contents and their everyday applications. So she encourages students to spend enough time talking about what they have already learned and to think of their meaning in real-life situations.

Mind-map is a good tool to communicate between the teacher and students. On the one hand, the teacher explains mathematical contents through a mind-map. On the other hand, students reveal what they have understood by drawing mind-maps. Although it is impossible to communicate directly with all students in the classroom, the teacher can develop a sense of what individual students understand and what they do not by simply looking at the mind-maps they draw.

(3) Characteristics of Assessment

Park2 tends to assess students' mathematical learning during the lesson, instead of spending extra time for evaluation:

I give stickers to students if they express their own thinking, approach to creative ideas, or help their peers. The number of the stickers comes to the score of performance assessment. In case students solve problems either in a worksheet or in a textbook, I ask a few questions in order to see they

understood the mathematical contents underlying the problems. If they are successful, I also give stickers to them. I always let students know what they have to understand in a given lesson and how many stickers they should receive. I also assess students' understanding by asking them to draw a mind-map per unit and to prepare a short report or to participate in a small project per several sections.

This kind of assessment aims at gathering evidence of students' understanding and informing them of what are the important contents they have to master in a given lesson. Note that Ms. Park2 also uses performance assessment in which students are required not merely to understand what they learned in school but also to know how it is applied to real-life contexts.

(4) Development of Lesson

Introduction	Development	Consolidation and Assessment
<ul style="list-style-type: none"> • Weather forecasting • Overview of contents across middle-school years. • Review of contents for the 2nd year in middle school. 	<ul style="list-style-type: none"> • Conditions of similar figures • Ratio of similarity • Various applications of similarity 	

4. Ms. Jang's differentiated lesson using real-life materials

Grade and Unit: 8-1, III. Equation and inequality

(1) Characteristics of Lesson Objectives

1) Objectives emphasizing on real-life applications

In the 7th national mathematics curriculum, there are six content areas - number and operation, geometry, variable and equation, measurement, probability and statistics, pattern and function - which should be taught in a way to think mathematically of various phenomena in everyday day life. The curriculum emphasizes the following three aspects. First, students should discover and understand mathematical principles and laws through various mathematical experiences. For example, students need to engage in the activities in which they interpret or connect basic concepts, principles, and laws. Students also need to have an experience of generalizing a phenomenon with various mathematical representations. Second, students should

be able to solve a problem in real-life situation by understanding it on the basis of mathematical knowledge and skills, making a reasonable and creative plan, and reflecting on the solution process. Third, students need to recognize the usefulness of mathematics and develop positive attitudes toward mathematics by solving various problems with mathematical knowledge and skills. Ms. Jang's instruction focuses mainly on the second and the third aspect. For example, she uses videotape capturing real-life experience for her lesson:

Let see. I'll show you my family's videotape last children's day. Listen carefully to the dialogue between Chi-Young and mother, and find the conditions that are necessary to solve the problem father asked. (She turns on VTR. Mother and Chi-Young do rocks-scissors-papers and pick acacia leaves.)

Mother (Teacher Jang): If you [Chi-Young] win, you can pick 2 leaves. If I win, I can pick only 1 leaf. We continue this game until we pick all the 21 leaves. Whoever collects more leaves wins the game.

Chi-Young (Teacher Jang's son): I see.

Mother: Let's count how many times we do rocks-scissors-papers.

Chi-Young: Mom, we did 15 times.

Mother: Good work, Chi-Young.

Did you understand? As you see, Mother and Chi-Young play 'pick leaves' game by doing rocks-scissors-papers. If Chi-Young wins, he gets 2 leaves. If mother wins, she gets only 1 leaf. There are 21 leaves all together and 15 times of rocks-scissors-papers before the game is over. Whoever has more leaves wins the game. Who do you think is the winner? (Note: Assume that we do not count the case of a tie in doing rocks-scissors-papers)

2) Teacher's recognition of lesson objectives

Ms. Jang develops problems for her lesson, which are different from those of textbooks or commercial workbooks. She sometimes shows a movie to the students and makes them solve a mathematical problem hidden in the movie. For an instance of the movie 'Die Hard', whether or not a bomb explodes depends on whether the students are able to solve the problem. Students then feel a kind of responsibility to solve the problem as if they were real actors in the movie.

Ms. Jang's mathematics problems are not limited to movie ones. As needed, all of her family members appear in problems. Students happen to enjoy these kinds of problems, because they know why they solve the problems.

I think that by doing mathematics students can learn two apparently incompatible attitudes or sprits, honesty and freedom. Mathematics is a discipline of definition. I believe that students can learn 'honesty' by engaging in logical thinking based on mathematically defined symbols and axiom systems. At the same time, students can learn 'freedom' by raising questions such as "why should I do in this way?" and by looking for different solution methods.

(2) Characteristics of Contents

Ms. Jang's lesson has some characteristics different from other teachers'. These include tracking students, considering their interest, and using real-life problems we hardly see in the textbooks.

1) Tracking students

Despite its advantages, tracking students imposes heavy burden on the teacher because it takes a lot of time to prepare for different lessons with instructional materials. As long as there are enough number of teachers and their willingness, nevertheless, Ms. Jang believes that tracking ultimately respects both students' right to learn and the teacher's right to teach:

Mathematical instruction based on tracking not only relieves teaching difficulties but also respects students. That is to say, the underlying principle of tracking is to protect students' right to learn. Students have the right to study through instruction tailored to their mathematical abilities. At first, students sometimes resist tracking. As times goes on, however, they recognize many merits of tracking. They come to have confidence in mathematics and be more active in expressing their ideas, mainly because there are peers whose mathematical abilities are similar. The teacher begins her lesson from where the students are, and thus students come to trust her more.

In Ms. Jang's school, students are divided into 4 classes in terms of quarterly written test scores. The classes are called as "Teon-Teon (sturdy)", "Eo-Teom" (prime), "Seol-Gi" (wise), and "Bae-Um" (learning) class, respectively. Classes are reorganized every quarter. Ms. Jang currently teaches an "Eo-Teom" class, 2nd top class of which students are from top 25% to 50%

in terms of test scores. So the differences of students' mathematical achievement are not much bigger. The instruction of the "Teon-Teon" class combines students' presentation of various solution methods, cooperative learning in small groups, and discussion in the whole class. However, the instruction of the "Eo-Teom" class requires the teacher's intervention. Of course, that of the other two classes needs more interventions such as the teacher's lecture and questioning with concrete materials.

In Ms. Jang's school, performance assessment according to students' different mathematical abilities is encouraged. It is ideal for the teacher to adjust the difficulty level of assessment in terms of students' ability. However, it is difficult for the teacher to develop not only instructional materials for regular lessons but also assessment items that are compatible with the differentiated instruction. Ironically, the teachers happen to teach students in terms of tracking but evaluate them by the same items. In short, assessment does not go with instruction.

If the teachers teach lower level classes repeatedly, they may have much burden to make different concrete materials. In Ms. Jang's school, every teacher deals with the four different classes throughout the year by exchanging the classes per quarter. Whereas this method gives the teachers an opportunity to teach different levels of students, it requires them to devise four different instructional materials and to reorganize their lesson plans.

2) Instruction for students' positive mathematical attitudes

Ms. Jang takes care of students' emotional states in her mathematics class. She wants her students to look forward to learning mathematics before the class, to enjoy learning mathematics during the class, and even to miss learning mathematics after the class. It is her philosophy on how to teach mathematics that instruction, like chatting with friends, should make students comfortable and interesting:

In order for her students to become more involved in class, Ms. Jang always has her students engaged in an activity called as the 'Math Golden Bell'. During the five minutes prior to the end of the class, Ms. Jang gives new problems to the students and they are to guess what the answers are. Students who are actively involved in the activities are given 'stickers.' The more stickers the students collect, the more opportunities the students are given to receive 'ddokbokki[students' favorite snack] certificates,' 'book certificates' or 'movie certificates.' The number of stickers students collect is reflected in their performance assessment as well, stimulating them to become more active in class.

Although it is said that most students study mathematics in order to get into a good university or to get high scores on tests, Ms. Jang believes that her students study mathematics because they have fun with it and therefore they actually have the desire to study it. Moreover, Ms. Jang devotes different kinds of efforts to help the students' understanding, including such strategies as the 'Math Class Weather Forecast' or 'Math Journal.' The 'Math Class Weather Forecast' is an activity that is done at the beginning of the class, having students express their emotional or physical status within 10 letters. This activity is especially effective in promoting a mutual understanding between the teacher and students.

For example, if I started with a mathematics class by writing 'Miserable Wednesday' on the upper left-hand of the blackboard, the students are bound to ask me "Teacher, what's the matter?". I playfully reply that I have menstrual cramps and the whole atmosphere of the class changes. Or, whenever I am walking through class looking over the students' work, I try to give encouragements, a warm shoulder patting, or ask "Why?" to students who express on their weather forecasts that they are sick either physically or mentally. Such simple gestures seem to make the students feel that they are being truly understood, and they actually start to look at school and class in a much more positive light. Such methods are also successful in detecting students who need counseling in early stages. Surprisingly enough, all of the impressions and emotions that students have during each math class are reflected through the ten-word long 'Math Class Weather Forecast.'

The Math Journal is used to record what happened during the class, what the students think about those incidents, particularly memorable lessons, feelings felt after each class, chats that were shared with friends in math class, and even things that the students had thought about in math class when they got bored; in such ways the Math Journal prompt the students to write everything that is related to the math class or the teacher, down to the very last detail. Ms. Jang explains that she first started with the Math Journals in the belief that, by thinking back on math class before going to sleep every night the students are able to remind themselves of what was learned in math class, and also that the Math Journals would promote communication between the teacher and students.

3) Mathematics problems that emphasize everyday life

Ms. Jang provokes students' attention by producing mathematical problem situation videos starring herself. Contrary to classes given by private institutes that are more or less focused on covering all content within a short amount of time, Ms. Jang differentiates her classes by looking for mathematics problems in everyday life, and also by providing different kinds of fun stories. Every now and then she also provokes concentration by conveying her expectations as a math teacher through songs. Students answered that they began to start thinking about why they need to learn mathematics because of such methods used by Ms. Jang. Moreover, the students said that there were hardly any students that fell asleep or dozed off because Ms. Jang's classes are always so interesting.

Whatever she does and wherever she is, Ms. Jang always tries to derive problem situations out of everything. She argues that, evidently math is important: thinking of lesson ideas in any place and time, math problems that can be found in daily life, or reversing one's thoughts to contort mathematics problems to real life or vice versa. However, the reason why she prefers such teaching methods is because she believes that the moment students start to understand how mathematics can be used in their everyday lives is when they naturally start to learn what mathematics really is. Ms. Jang explains as follows.

Even when I am playing games with my child at home, when I am watching a movie, or when I am out grocery shopping, I strive to constantly ask my son questions and when he responds, I think of whether or not such situations can be applied as problem situations. Once I feel that they could be incorporated into math problems, I ask my husband to record the situation on videotape. I sometimes present these videotapes during class, or derive new problems out of them.

As one example, Ms. Jang explains how she had presented famous art pieces and explained why she liked them in the course of carrying out an interdisciplinary course between mathematics and art. Using a projection TV, she showed various masterpieces by world famous artists and took the example of a museum and finally raised a mathematics problem related to museum entrance fees. She created problems by differentiating children's fees and adults' fees. While a conversation between her husband and son is heard in the background over the audio, the students try to relieve her son Chi-Young of his curiosities related to the museum entrance fees. After solving the problem, she has the students find an answer that is most relevant to the meaning of the problem.

As she always emphasizes a class filled with fun themes and everyday applications, she is able to discipline the students and attract their concentration without having to be too stern with them. That is one of the very unique characteristics of her class. If any of the students start talking in class, rather than giving them any punishments, she controls them with a short and strong gaze. Her classes were characterized by fun stories, themes drawn from the students' lives, and her unique charisma.

(3) Characteristics of Teaching and Learning Methods

1) Drawing students to get used to accurate problem solving

After the students solve simultaneous equations, Ms. Jang leads her class by drawing her students to correctly and accurately understand the 5 steps of solving mathematics problems in the following fashion. She asks students about each step of the problem solving process, the students answer, and then she provides a brief supplementary explanation. By going through this routine continuously, she has students get ample opportunities for practice. She always adds new questions to almost all explanations and demands straightforward answers from the students. There are some differences with other teaching methods in terms of questioning and answering, but she explains that she utilizes such a method because it is incorporated particularly in the full understanding part of the class.

2) Problem presentation using various methods

After presenting problems through role-play activities, presenting problems related to the video contents, etc., Ms. Jang prepares worksheets that contain problems connected with the activities. Through role-play with these problems, the lesson is carried out using the 5-step problem solving process noted earlier.

(4) Characteristics of Assessment

Assessment is significant criteria to grasp to what extent students have achieved learning goals, and to confirm whether or not the students are prepared to advance into the next learning point. Results of assessment are usually used for discerning students who are in need of individual guidance, and also as results of performance assessment. Ms. Jang stresses the fact that the results of assessment are important reflective data to figure out whether her classes have been appropriately applied to each student's learning.

In addition, Ms. Jang argues that the current assessment system, which is focused on the objectivity of assessment, influences teaching methods and teaching contents as well. Therefore regardless of the many endeavors to design new methods and contents, if assessment methods centered on multiple-choice and short answers are demanded in order to eliminate subjectivity, teaching methods and classes cannot dramatically change owing to pressure posed by parents and students. Ms. Jang believes that lessons and classes can change only if assessment is changed.

1) In-class assessment

In this assessment method, the teacher has students present their math journals that are used to improve their attitudes towards mathematics, and assess how much each student has progressed. In consideration that mathematical skills and mathematical communication are becoming more and more in demand, Ms. Jang explains that this method is very effective. Such assessment through presentation not only helps students express their thoughts on the mathematical contents they have learned, but also allows them to find such mathematical contents in their real lives and appropriately apply them.

2) Formative Assessment

Most students taking Ms. Jang's classes are interested in mathematics. The school that Ms. Jang teaches at has adopted the system where students move to whichever level class they belong to, and the current classroom at hand is classified as an "Eo-Teom" class. The students are sorted into "Teon-Teon" class, "Eo-Teom" class, "Seol-Gi" class and "Bae-Um" class according to their mid-term scores, in that order. Needless to say, the current class may seem to be more attentive simply owing to the fact that it is classified as the "Eo-Teom" class. However, it is easily observed that other students in other levels are equally interested and attentive during class. Although other teachers argue that this area of classes is hard to lead in an interesting and fun way, Ms. Jang counter-argues that any area can be led in an interesting way if only teachers are fully prepared and can induce problem situations in an intriguing fashion.

(4) Development of Lesson

1. Introduction	Development	Consolidation and Assessment
<ul style="list-style-type: none"> • Opening to mathematics 	<ul style="list-style-type: none"> • Motivation and Problem Presentation 1 • Problem Solving 1 • Motivation and Problem Presentation 2 • Problem Solving 2 • Teachers' Instruction and Problem Solving 	<ul style="list-style-type: none"> • Golden Bell*

* Golden Bell: Once the teacher provides a problem, each student writes his or her answer on a piece of paper in large handwriting. Following the teacher's explicit directions, the students hold up their answer sheets and solve the problems. The name Golden Bell comes from the name of a popular program aired on national TV. In the program, 100 high school students are given writing material and hardboard panels to solve 50 problems. Those students who solve all 50 problems are given the opportunity to ring the big Golden Bell and receive a reward.

5. Ms. Kim²'s various activities for motivation in class

Grade and Topic: 9-1, I. Square Roots and Real Numbers

(1) Characteristics of Curriculum and Contents

Ms. Kim seeks contents that can be applied to real life and incorporates them into her lessons or recomposes different problem situations so that they respond to current events, circumstances surrounding the school, or students' latest interests and applies them to her lessons. Despite such justification, Ms. Kim stresses that "Teachers must carefully set up the problem situation so that they can attract the attention of students and must also introduce teaching methods that are based on the unique characteristics of students."

Ms. Kim does not simply follow the contents of the textbook but recomposes the order, presentation, and material of the textbook contents in her own creative way. She also uses a supplementary material called 'Math Reading' that she personally designed.

(2) Characteristics of Teaching and Learning Methods

1) Assignments by level and worksheet preparation

Ms. Kim claims that although most classes are carried out upon the textbooks that were composed based on the curriculum, the actual curriculum, concrete class contents and directions

that are executed in classrooms have failed to reach such levels. As a solution to this problem, Ms. Kim suggests to put more thought into classes and manages her classes by providing easier questions and answers and repetitive learning of basic issues and ideas for less progressed students, and separately assigning homework via the Internet for higher level students.

Ms. Kim starts out class by going through a short recap of what was learned in the previous class, and then explains what the present day's contents would be relevant to. Ms. Kim explains that each year's students are never the same, and therefore believes that her classes should be different every year. Upon such belief, she constantly makes the effort to experiment with diverse teaching methods. In order to provide students with ample opportunities and activities through which they can learn mathematical material, she utilizes computers in her lessons. The advantage of using computers in class is that computer-based lessons can be elongated to studying at home and thus students can independently carry out their own study. In particular, such computer-based methods are useful in identifying shy students who are reluctant at raising their voices in class. The shier students can actively utilize the class web page and retrieve material and furthermore create their own material and upload the material onto the web page. Material uploaded by students onto the web page also served as good evidence as to the students' creativity and individuality.

2) Learning activity centered classes

Ms. Kim's classes are centered on students' activities. Lesson planning directions are designed around answers given to questions presented by the teacher, and worksheets are composed to make cooperative learning available. PR Learning is carried out once every week by students recomposing the contents provided by the teacher and then the students presenting the recomposed contents within 5 minutes. Evidently students have a tendency to become disoriented or rowdy in the course of preparing for group presentations due to vigorous debates and discussions, but Ms. Kim believes that her classes should always allow liberal opinion presentation, participation and communication.

3) Cooperative learning by groups

The reason to consistently maintaining cooperative learning by groups is based on the conception that knowledge can be more thoroughly stored and used when students are exposed to the process of mutual questioning, rather than when students are passively fed with

knowledge provided by the teacher without considerable effort. Despite such positive aspects, the reason why cooperative learning by groups is not executed in most classes is because teachers believe such teaching atmosphere provokes students to become ill disciplined. However Ms. Kim counteracts to such concerns by rewarding appreciable compensation to student groups who excel in cooperative activities or display remarkable progress in their improvement scores that are given according to individual competence, and therefore can smoothly manage her cooperative learning classes.

Moreover, in the perspective of cooperative learning, by managing leveled classes in the form of cooperative learning, Ms. Kim asserts that she enables classes that allow all students opportunities to participate, classes that facilitate amicable communication in the course of mutual questioning, answering, and reaching conclusions, and therefore can derive humanistic educational effects that recognize each other's strengths, weaknesses and individualities. In particular, students are exposed to an atmosphere that promotes listening to others' opinions, which is a crucial factor for contemporary day students who are only capable of expressing their own opinions and not listening to others, and that allows students to access other various problem solving techniques provided by peers and not by the teacher.

4) Classes utilizing real life subject matter

Ms. Kim makes extra efforts to manage her classes using real life subject matters that enable students to have hands-on experiences that "are as directly connected to students' real lives as possible, so that they can think 'oh, that!' whenever they feel mathematics has been applied to their lives." In particular, the perspectives students have on certain objects or events will go through a dramatic change when they are exposed to such classes on a long-term basis. Through such classes students will obtain an interdisciplinary understanding of mathematics and other subjects, and also will be able to nurture their logical thinking abilities.

The main reason that most students perceive mathematics to be harder than other subjects lies in the fact that they believe mathematics is no more than the study and learning of a mental product that is entirely irrelevant to real life. Therefore the teacher is responsible to demonstrate for students how it can be incorporated into real life so that they can confirm how mathematics is used. In other words, biases that students tend to have related to mathematics should be eliminated in order for them to find a new attraction and interest in it.

Continuing to apply real life to mathematics class inspires students to adopt a new perspective on everyday objects and events. I also feel that students obtain the power to apply, fully understand and logically assess when other subject matters are linked with mathematics. In addition, most assessment of classes using real life are executed with performance assessment; if real life is truly appropriately incorporated, students are able to nurture their ability to apply, consequently allowing students to reach higher achievements in terms of the performance assessment. Performance assessment using real life usually deals with several mathematical areas and therefore students are also able to cultivate their mathematical powers. For example, through dealing with math problems that require knowledge of probability, binomial equations and inequalities in preparing math reports using the World Cup, students will be able to look at the World Cup in a wholly different light.

Ms. Kim's classes that use real life do not merely focus on theories and ideas, and allow students to find their own subject matters in their real lives so that their real lives can be incorporated into mathematics class. As a brief and representative example, her classes require students to, not only explain ideas related to functions but also provide concrete examples as to how functions can be applied to or found in real life.

Ms. Kim carries out her classes by "personally developing fun and interesting teaching methods", using real life situations that are easily found in our lives and creating problems to be used during class. When asked how she deals with units that cannot be easily adapted to everyday life, Ms. Kim flatly replied that "I guess I would not be able to use real life in those cases but I don't think those cases exist in math." Ms. Kim believes that, as long as the teacher does extensive thinking and research, all mathematical situations can be found in real life, for example while watching TV or cooking. Additionally, Ms. Kim argues that problematic issues can be found, not only in level differences between students, but also in the current curriculum. In most cases, the current curriculum defines contents that are based on real life to fall in the category of 'advanced learning' and therefore a large number of teachers believe that classes based on real life are only required in advanced classes. On the contrary, classes that utilize real life easily provoke motivation and interest in students and foster applying abilities. Moreover, students verbally express their surprise and interest in unpredicted mathematical facts that can be directly related to real life and also nurture their curiosity by looking for more examples.

Such classes stimulate the students to go out and find their own examples of mathematical application in real life.

5) ICT-based learning

Although the fact that students can approach mathematics with more interest when using ICT in class is most welcoming, Ms. Kim does not believe that each and every detail in the textbook can be taught using ICT content. Nevertheless, Ms. Kim does argue that the ICT data that were introduced into the classroom should also be incorporated into assessment so that content that is hard to assess solely through exams can be covered. As a result of such assessment, Ms. Kim testifies that not only was she subject to indirect experiences that provoked her improve her teaching methods, but also that the quality of the results improved with each performance assessment.

For example, I set up the bulletin board so that instant replies can be posted, and so I can offer my comments and give out scores to the students' products. Other students can also post their opinions and comments. As a result, students were able to immediately access their assessment results, and naturally peer review was also made possible. Students can realize that 'he/she excels in these aspects,' or 'I could improve in those aspects', and compare their scores and results with those of other peers and analyze exactly why they were given such comments and assessment. I don't have to make the extra effort for students to share their results, nor do I have to explain to the students about my assessment.

Ms. Kim particularly stresses that instantaneity and objectivity in terms of assessment are crucial factors in order to achieve efficiency in carrying out performance assessment using ICT. In other general cases, Ms. Kim gives one-on-one feedback to students by making assessment sheets made out of worksheets that include the use of riddles or puzzles. In other words, the answers to the worksheets can only be derived through riddles or some sort of coloring, and then the teacher would know whether or not the students had correctly solved the questions and problems and finally gives the students feedback. Additionally, as content learning and assessment are conducted at a similar ratio, 60% of the class period is devoted to learning content and the remaining 40% is devoted to assessment.

Self-composition by students is another assessment method that Ms. Kim utilizes. Such assessment has the aim of assessing to what extent the students have understood lesson contents and also of rearranging subject matter that had been dealt with in class. Ms. Kim focuses on assessment as a different form of learning, not on assessment merely as a tool for evaluation. Results from such assessment are used in individual learning.

(3) Characteristics of Assessment

PR classes are led by a group of students who are responsible to produce material that can show what has been learned on the day in question, and lead the class by presenting the material to the rest of the students.

(4) Development of Lesson

1. Introduction	2. Development	3. Consolidation and Assessment
<ul style="list-style-type: none"> • Review of Previously Learned Contents • Presentation of Learning Goals 	<ul style="list-style-type: none"> • Motivation • Mind Opening • Thought Refining • Building/Making • Learning Through Presentation • PR Learning 	

IV. Conclusion

In actuality, most middle school mathematics classes are carried out in lecture forms. High schools in particular, generally take on lecture-style mathematics classes. However, as is seen in the above-mentioned examples, many various teaching methods are being experimented in order to derive student-centered understanding within the classroom. For example, unlike the traditional teaching method of teacher-centered instruction, the current teaching methods utilize concrete manipulative, encouraging students to do their own independent research according to subject matter, consequently fostering a profound awareness of mathematical contents. In addition, access to the Internet, networked computers and projection TVs that are provided in all elementary, middle and high school classrooms are used for students to view various multimedia with the aim of motivating students, fostering understanding by accurately showing

the contents of the lesson in the course of lesson development, have the students easily share their finished assignments with peers, and allow a systematic presentation of lesson conclusions. Such consecutive class activities and their conclusive products are reflected in each student's performance assessment so that uniform assessment can be avoided and to move towards using more diverse assessment methods.

Abacus classes can be a great source of learning. Teaching through a step by step approach of getting an answer. Clearing the doubts that a child asks in the learning process. Understanding his/her abilities and setting the difficulties on those parameters. Conclusion. We hope some of these approaches and strategies of teaching mathematics will effectively improve the dynamics of a classroom. Teachers and parents must not worry about how to teach maths. Rather they must focus on how to teach maths in an interesting way. mathematics classrooms the answers are always known and this offers students little opportunity. for creativity and discovery. Such a view. of. mathematics classrooms suits the preferred learning. style. of. Styles. in. Secondary Classrooms. The. single-sex. boys'. class was taught in a very traditional and separate knowing way with. the. boys. Teaching - Secondary Math Classroom. Collection by Andrea Cook. 369. Teaching Resources Math Classroom Teaching Learning Math Math Lessons Education Math Middle School Math Math Resources Math Activities. 70+ Awesome Websites for Teaching and Learning Math. This isn't just any list of the best websites for teaching math. It's the BIGGEST list of the best websites for teaching math. Bookmark it! Free Standards for Mathematical Practices posters for your classroom. Perfect for the upper elementary or middle school classroom. Secondary Math Teaching Secondary Radians Secondary Math Classroom Teaching Open Letter Letter To Students Dear Students Degrees. Degrees vs. Radians. on secondary school students' achievement in Mathematics using innovative models teaching approach. The paper describes innovation and modeling for improving teaching and learning of Mathematics that will address the barriers which make it difficult to achieve numeracy skills. The strategies are offered as a means of evaluating and aligning students' activities to promote stronger learning of Mathematics. A typical classroom learning may not encourage students' participation, self expression and may not build the required involvement level on the part of the student. There is a great need to improve quality of teaching in Mathematics. This can be possible by bringing fundamental changes through innovative approaches. www.iosrjournals.org. 6 | Page. teaching-secondary-mathematics-texbook baru.pdf - Free ebook download as PDF File (.pdf), Text File (.txt) or read book online for free. Teaching secondary mathematics: techniques and enrichment units. -Ninth edition | Alfred S. Posamentier, The City College, Mercy College, New York, Beverly S. Smith, The City College, The City University of New York. pages cm Includes bibliographical references and index. School Classroom 309 Cross-Catalogue of Enrichment Units 310 1 Constructing Odd-Order Magic Squares 318 2 Constructing Even-Order Magic Squares 320 3 Introduction to Alphametics 324 4 A Checkerboard Calculator 327 5 The Game of Nim 330 6 The Tower of Hanoi 333 7 What Day of the Week Was It?