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Failure of the cluster decomposition principle in QCD

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The cluster decomposition principle is used to derive Ward-Takahashi-like identities in QCD. Comparison with published calculations of QCD vertices finds that these new identities are over-constrained Slavnov-Taylor identities, thus demonstrating by contradiction the failure of the cluster decomposition principle in the perturbative and far-infrared regimes.

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The cluster decomposition principle (CDP) states that as two particles separate their wavefunctions become mutually independent. This is intuitively expected unless the theory in question is confining. This paper proves, in both the perturbative regime and the infrared limit up to a small caveat, that the CDP fails in a confining theory.

The proof is by contradiction. It begins with BRST invariance and derives identities among the Green's functions in almost the standard way, but without the introduction of sources for the composite operators in the BRST transformations. Instead, Green's functions with composite operators are broken down into sums of products of connected Green's functions that do not contain composite operators. This is possible because the CDP restricts connected momentum space amplitudes to only one momentum space delta function, as clearly explained in [1]. An identity between the gluon and ghost propagators can be used to factor the propagators out of the identities among the three-point Green's functions.

The resultant vertex identities are quite straightforward, easy to solve, and hold at the tree level but the crux of this letter is that they are *wrong*, which is demonstrated in both the perturbative regime and the infrared limit by substituting in previously obtained results ([2, 3, 4, 5, 6] *and references therein*). Since assuming the cluster decomposition principle leads to a false conclusion (incorrect identities) the principle must be wrong. Indeed, it will be argued from the form of the following identities that imposing the CDP overconstrains the theory.

If the gluon, ghost and anti-ghost fields are given by $\vec{A}_\mu, c, \bar{c}$ respectively, then the QCD action respects the BRST identities

$$\delta_B\vec{A}_\mu \;=\; \theta\vec{D}_\mu c,\;\; \delta_B c^a=\theta\frac{1}{2}gf^{abc}c^bc^c,\;\; \delta_B\bar{c}^a=-\theta\frac{1}{\xi}\partial_\mu\vec{A}^a_\mu, \tag{1}$$

where θ is a Grassman parameter. The obvious starting point for deriving Green's function identities is the propagators,

$$\begin{aligned} 0 &= \langle\delta_B(\vec{A}_\mu\bar{c})\rangle=\theta\partial_\mu\langle c\bar{c}\rangle+g\theta\langle(\vec{A}_\mu\times c)\bar{c}\rangle-\frac{\theta}{\xi}\partial_\nu\langle\vec{A}_\mu\vec{A}_\nu\rangle \\ &= \theta\partial_\mu\langle c\bar{c}\rangle_C-g\theta\langle\vec{A}_\mu\rangle_C\times\langle c\bar{c}\rangle_C+g\theta\langle\bar{c}\vec{A}_\mu\rangle_C\times\langle c\rangle_C-\frac{\theta}{\xi}\partial_\nu\langle\vec{A}_\mu\vec{A}_\nu\rangle_C \\ &= \theta\partial_\mu\langle c\bar{c}\rangle_C-\frac{\theta}{\xi}\partial_\nu\langle\vec{A}_\mu\vec{A}_\nu\rangle_C, \end{aligned} \tag{2}$$

where the subscript $_C$ denotes a connected amplitude. The second line follows from the CDP because the components of a composite operator, like that in the middle term of the first line, are linked by a momentum space delta function. As noted above, the CDP forbids any connected Green's function from containing more than one delta function.

The result is a relationship between the ghost renormalisation and the longitudinal gluon renormalisation. Eq. (2) does not include the gluon polarisation because of the momentum factor in front of the gluon propagator. This relationship is essential for extracting the vertex identities from those of the Green's functions.

To derive an identity between the ghost propagator and the ghost-gluon vertex one starts with

$$\begin{aligned} 0 &= \langle\delta_B(c^a(y)\bar{c}^b(z)\bar{c}^c(x))\rangle \\ &= \theta\Big(\frac{1}{2}g\langle(c^dc^e)(y)\bar{c}^b(z)\bar{c}^c(x)\rangle f^{ade}+\frac{1}{\xi}\partial^\mu_x\langle c^a(y)\vec{A}^c_\mu(x)\bar{c}^b(z)\rangle-\frac{1}{\xi}\partial^\mu_z\langle c^a(y)\vec{A}^b_\mu(z)\bar{c}^c(x)\rangle\Big) \\ &= \theta\Big(-g\langle c^d(y)\bar{c}^b(z)\rangle_C\langle c^e(y)\bar{c}^c(x)\rangle_C f^{ade}-\frac{1}{\xi}\partial^\mu_x\langle c^a(y)\vec{A}^c_\mu(x)\bar{c}^b(z)\rangle_C+\frac{1}{\xi}\partial^\mu_z\langle c^a(y)\vec{A}^b_\mu(z)\bar{c}^c(x)\rangle_C\Big), \end{aligned} \tag{3}$$

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where the final step employs the CDP. Eq. (2) can now be used to truncate the Green's functions in (3) yielding the momentum space vertex identity

$$0 = -g\langle c\bar{c}\rangle_C^{-1}(q^2)f^{abc} - ip_\mu\frac{1}{\xi}\Gamma^\mu(q,-p-q)^{acb} - i(p+q)_\mu\frac{1}{\xi}\Gamma^\mu(q,p)^{abc}, \tag{4}$$

to which one solution is

$$\Gamma^\mu(p,q) = igq^\mu G(q^2)f^{abc} + i[p^\mu(q^2+p\cdot q) - (p+q)^\mu p\cdot q]T_G(p^2,q^2,p\cdot q)f^{abc} \tag{5}$$

where the ghost propagator is

$$\langle c\bar{c}\rangle_C^{ab}(p^2) = \frac{\delta_B^{ab}}{G(p^2)p^2}. \tag{6}$$

Comparing with the ghost-gluon vertex calculated in [2] it is immediately clear that (5) is not correct. Of course (5) may not be a unique solution, but substituting the vertices and ghost propagator found by Davydycheff *et. al.* finds that (4) is not satisfied. From eq. (C.2) in [2] we have

$$G(p_1^2) = \frac{g^2\eta}{(4\pi)^{n/2}}\frac{C_A}{4}[n-1]\kappa(p_1^2), \tag{7}$$

in Feynman gauge, where n is the spacetime dimension, η is given by

$$\eta = \frac{\Gamma^2(n/2-1)}{\Gamma(n-3)}\Gamma\left(3-\frac{n}{2}\right), \tag{8}$$

and κ by

$$\kappa_i \equiv \kappa(p_i^2) = -\frac{2}{(n-3)(n-4)}(-p_i^2)^{\frac{n-4}{2}}. \tag{9}$$

To simplify the analysis the ghost-gluon vertex given by eq. (D.1) in [2] is restricted to the on-shell case $p^2 = 0 = (q+p)^2$. The vertex contribution to (4) is then

$$\begin{aligned}
&-(q+p)_\mu\frac{1}{\xi}\Gamma^\mu(q,p)^{abc} - p_\mu\frac{1}{\xi}\Gamma^\mu(q,p+q)^{abc} \\
&= -\frac{g^2\eta}{(4\pi)^{\frac{n}{2}}}\frac{C_A}{4(q^2)^2}\Big(p\cdot q(q^2)^2[2q^2\phi+\kappa_1-\kappa_2-2\kappa_3] - (p+q)\cdot q(q^2)^2[2q^2\phi+\kappa_1-2\kappa_2-\kappa_3] \\
&\quad -q^2q\cdot p[q\cdot(q+p)(2\kappa_1-\kappa_3)+p\cdot q\kappa_2] + 2q^2p\cdot q[q^2p\cdot q\phi - q^2\kappa_1 - q\cdot p\kappa_2 + q\cdot(p+q)\kappa_3] \\
&\quad +q^2p\cdot q[q^2\kappa_1+p\cdot q(\kappa_2+\kappa_3)-2q^2p\cdot q\phi] + q\cdot p\,q\cdot(q+p)[2p\cdot q\,q\cdot(q+p)\phi - 3q\cdot(q+p)\kappa_1 - 3p\cdot q\kappa_3] \\
&\quad -(q\cdot p)^2[-2(p\cdot q)^2\phi + 3q\cdot p\kappa_1 + 3p\cdot q\kappa_2]\Big).
\end{aligned} \tag{10}$$

The reader who needs convincing that these expressions fail to satisfy (4) since (10) is very complicated, need look no further than the linear dependence of $G(p^2)$ on n. This has no counterpart in (10) and therefore does not cancel from (4).

Since the CDP is the only additional assumption to BRS invariance, the failure of the resulting identities is proof that it does not hold. The above argument could be repeated for the triple gluon vertex but that would serve little purpose as both the identities and the vertex are more complicated, expressing the same result less clearly.

Rather, observe that the proper function identities derived using the CDP are restricted versions of the Slavnov-Taylor identities. Consider the Green's function identity derived from

$$\begin{aligned}
0 &= \langle\delta_B A_\mu^a(x)A_\nu^b(y)\bar{c}^c(z)\rangle \\
&= {}_x\partial_\mu\langle c^a(x)A_\nu^b(y)\bar{c}^c(z)\rangle + g\langle A_\mu^d(x)A_\nu^b(y)\rangle\langle c^e(x)\bar{c}^c(z)\rangle f^{ade} \\
&\quad + {}_y\partial_\nu\langle c^b(y)A_\mu^a(x)\bar{c}^c(z)\rangle + g\langle A_\mu^a(x)A_\nu^d(y)\rangle\langle c^e(y)\bar{c}^c(z)\rangle f^{bde} - \frac{1}{\xi}\,{}_z\partial_\rho\langle\delta_B A_\mu^a(x)A_\nu^b(y)A_\rho^c(z)\rangle
\end{aligned} \tag{11}$$

Repeating the above procedure and using eq. (5) finds

$$(p+q)^\lambda\Gamma_{\mu\nu\lambda}(q,p) = J(q^2)(g_{\mu\nu}p^2 - p_\mu p_\nu) - J(q^2)(g_{\mu\nu}q^2 - q_\mu q_\nu), \tag{12}$$

where $J(p^2)$ is the polarisation of a gluon with momentum p, in keeping with the notation of [2, 3, 5]. It is instructive to compare it to the STI for the triple gluon vertex

$$(p+q)_\lambda \Gamma^{\mu\nu\lambda}(q,p) \;=\; \tilde{\Gamma}^{\nu\lambda}(q,-p-q)J(q^2)(g^{\mu}_{\lambda}q^2 - q_\lambda q^\mu)G((q+p)^2) - \tilde{\Gamma}^{\mu\lambda}(p,-p-q)J(p^2)(g^{\nu}_{\lambda}p^2 - p_\lambda p^\mu)G((q+p)^2), \tag{13}$$

where the ghost-gluon vertex in Landau gauge is given by

$$\Gamma^\mu(p,q)^{abc} = -igf^{abc}q_\rho \tilde{\Gamma}^{\rho\mu}(p,q). \tag{14}$$

Substituting eq. (5) into (13) finds (12) thus indicating that the former is a more general version of the latter.

The differential Ward identities found in the limit $q \to p$ illustrate this point even further. If $Z_3, \tilde{Z}_3$ are the gluon and ghost field renormalisation respectively, and $Z_1, \tilde{Z}_1$ are the triple gluon and ghost-gluon vertex renormalisation respectively, then taking the STI (13) in this limit yields

$$\frac{Z_3}{Z_1} = \frac{\tilde{Z}_3}{\tilde{Z}_1}, \tag{15}$$

but applying it to (4) and (12) produces

$$\frac{Z_3}{Z_1} = 1 = \frac{\tilde{Z}_3}{\tilde{Z}_1}, \tag{16}$$

which is again a special case of the true relation. The logical conclusion is that imposing the CDP overconstrains the proper functions and forbids the true vertices as solutions.

The proof so far is limited to perturbation theory because the true vertices used to disprove the identities were calculated perturbatively. That the CDP fails at one-loop is enough to say that it fails at arbitrary loop order, but perturbation theory fails completely when the coupling constant g becomes sufficiently large and the vertices must be calculated in some other way. Furthermore the most important physics of QCD such as confinement and chiral symmetry breaking occurs in the far infrared region to which we now turn.

The dependence on momentum of the gluon and ghost propagators and their interactions in the far infrared limit [4, 6] has been found solving Dyson-Schwinger equations (DSEs) together with anomalous dimensional considerations. The gluon and ghost propagators vary as

$$Z_3(p^2) \propto (p^2)^{-2\kappa}; \;\; \tilde{Z}_3(p^2) \propto (p^2)^{\kappa}. \tag{17}$$

where κ is a positive [7] constant argued to be in the range $0.5 \leq \kappa < 0.6$ [8, 9] while the vertices vary as

$$\Gamma^{(n,m)} \propto (p^2)^{n-m}, \tag{18}$$

where $2n$ is the number of ghost legs and m is the number of photon legs. For gluon vertices, such as the ghost-gluon vertex encountered already, the momentum p_μ factors off in Landau gauge and it is the $\tilde{\Gamma}$ vertex that is inserted into (18). This illustrates the interesting but long-known fact [4, 10] that the three-point ghost-gluon vertex regains its tree level momentum dependence in the far infrared. Unfortunately for the CDP, the dressing of the gluon propagator does not.

Caveat: The proper function infrared dependencies presented in these last two equations are known to be self-consistent solutions to the DSE, but they are not known to be unique. Other solutions might exist for which this argument no longer holds.

A phase transition might restore the CDP at or above the critical temperature where deconfinement is expected. If the correlation between confinement and failure of the CDP is strong, equations (4,12,16) might hold in the deconfining phase. A finite-temperature analysis might be interesting.

The failure of the CDP has been demonstrated in two important regimes, the perturbative and the far infrared. The perturbative region is important because it is easiest to calculate things there. It is also worth noting that some phenomena relevant to confinement, such as the magnetic and other condensates (*eg.* [11, 12, 13, 14, 15]), are manifest even at one-loop. Further considerations have indicated that imposing the CDP is to impose a false assumption that overconstrains the proper functions. Restoration at zero temperature in some intermediate momentum range has not been ruled out although it seems unlikely. Finally, the possibility that the CDP might be restored above the critical temperature and that satisfaction of the failed identities derived in this paper might be a signature for the deconfining phase has been discussed.

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The cluster decomposition principle is one of the funda-mental principles in quantum eld theory [1]. In its general form, the cluster decomposition principle states that in a certain limit, a correlation function involving many quantum operators can be decomposed into products of correlation functions involving a smaller number of operators. This principle, and the possibility that it might fail, have played a signicant role in theoretical physics. On one hand, this principle has been widely used to constrain the form of the scattering matrix in eld theory [2â€“4].Â This observation has motivated signicant efforts seeking to prove the failure of this principle in QCD. All these previous works on the cluster decomposition principle, however, are purely theoretical. The Cluster service is shutting down because quorum was lost. This could be due to the loss of network connectivity between some or all nodes in the cluster, or a failover of the witness disk. Run the Validate a Configuration wizard to check your network configuration. If the condition persists, check for hardware or software errors related to the network adapter. The Cluster service cannot be started. An attempt to read configuration data from the Windows registry failed with error '2'. Please use the Failover Cluster Management snap-in to ensure that this machine is a member of a cluster. If you intend to add this machine to an existing cluster use the Add Node Wizard. Alternatively, if this machine has been configured as a member of a cluster, it will be necessary to restore the missing configuration data that is necessary for the Cluster Service to identify that it is a member of a cluster. Perform a System State Restore of this machine in order to restore the configuration data. Event ID: 7024. The Cluster Service service terminated with service-specific error The system cannot find the file specified.. Event ID: 7031.