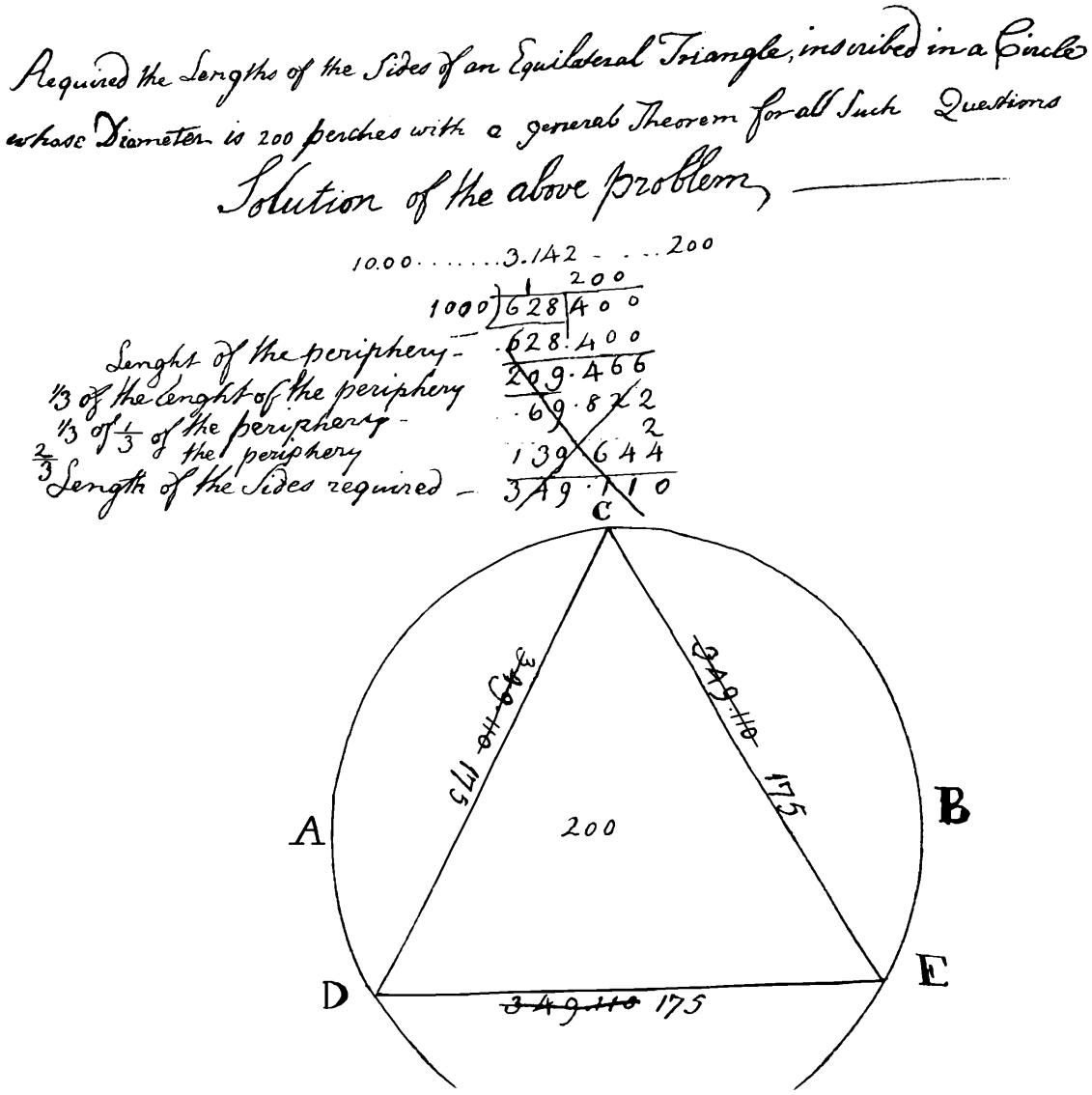
Benjamin Banneker's Mathematics – In His Own Handwriting

John F. Mahoney mahoneyj@sidwell.edu
Benjamin Banneker Academic High School www.benjaminbanneker.org
800 Euclid St., NW Washington, DC 20001

Introduction

Benjamin Banneker, the son of a former slave, was born in 1731 on a farm near Baltimore. From books loaned by a neighbor, Banneker taught himself surveying, astronomy, and mathematics. He later published several almanacs containing his astronomical observations. These almanacs widely distributed in Pennsylvania, Delaware, Maryland and Virginia. In 1791, Banneker received an appointment to assist in the survey of the boundaries of the Federal Territory - a ten-mile square now known as the District of Columbia. Banneker was also a social activist; he wrote a long letter to Secretary of State Thomas Jefferson likening the slavery of Negroes in the US to the enslavement of the American Colonies by the British. He attached his first Almanac as evidence that an African-American could be a distinguished scientist. Banneker died in 1806 on his Maryland farm. Banneker was honored on a 1980 US postage stamp.

In his journ Banneker wrote and collected mathematical puzzles written in verse. These journals served as his notebooks for astronomical observations, his diary, and his math notebook. Unfortunately only one of his journals survived a fire on the day of his funeral. The mathematics in this journal consisted of six puzzles and two pages of mathematical writing. Banneker's six puzzles from the journal were published in an excellent biography of him written by Silvio Bedini. To my knowledge, Banneker's mathematics in his own handwriting has never been reproduced. I located a microfilmed copy of his journal at the Maryland Historical Society in Baltimore. The quality of the reproduction was poor, but I was helped by Mr. Omar Rumi of Kuala Lumpur, Malaysia and my son, Quinn, a student at MIT. With the combination of Banneker's excellent penmanship, quality scanning and most of all painstakingly accurate photographic retouching on the part of Mr. Rumi, I am able to reproduce Banneker's actual handwriting of his mathematics.



Required the Lengths of the Sides of an Equilateral Triangle inscribed in a Circle whose Diameter is 200 perches, with a general Theorem for all such Questions

Solution of the above problem

10.00 ….. 3.142 …… 200

200

1000)628400

Lenght of the periphery	628.400
1/3 of the length of the periphery	209.466
1/3 of 1/3 of the periphery	69.822
	2
2/3 of 1/3 of the periphery	139.644
Length of the Sides required	349.110

Note: In Banneker's journal there is a big X crossing out the last five numbers in his calculations and adjacent to each of the 175's in his figure is ~~349.110~~

This example deals with the problem of finding the lengths of the sides of an inscribed equilateral triangle in a circle of diameter 200 perches. A perch, a synonym of a rod, is a unit of measurement equal to 16 ½ feet. There are 320 perches in a mile. A surveyor could find the number of acres in a rectangular piece of land by multiplying the length in perches by the width in perches and dividing the product by 160.

What did Banneker do to solve this problem? First of all he calculated the circumference, which he calls periphery, by multiplying an approximation of π (3.142) by 200. He did this by multiplying 3142 by 200 to get 628400 and then dividing that by 1000 to get 628.400 (How often have do we, as teachers, pull our hair out when we see our students multiplying or dividing by powers of 10?) He then finds 1/3 the length (misspelled in his journal) of the circumference and later 2/9 of the circumference. He adds those numbers together to get 349.110, but clearly he knew that the side of the equilateral triangle had to be less than the diameter. He crossed out his work and then labeled the sides of triangle 175 which is approximately half of 349.110.

Using the properties of a 30º-60º-90º triangle and a circle of radius 100, one can see that the length of the side of the equilateral triangle is 173.205 which shows that Banneker's solution is within 1% of the actual one. The side of the equilateral triangle is $\sqrt{3}$ times the radius of the circumscribed circle.

How did Banneker figure this out without using the properties of a 30º-60º-90º triangle or trigonometry? Banneker calculated 5/9 of the circumference to get 349.110 and by taking half of that he essentially computed 5/18 of the circumference. Since $\frac{5}{18} \cdot 2\pi \cdot 100 \sim 174.533$ Banneker's method is quite good. It would have been even better if he had taken exactly half of 349.110 to get 174.555. Banneker's method essentially uses $\frac{5}{9}\pi$ to approximate $\sqrt{3}$. Perhaps this approximation was a rule of thumb that surveyors used in the 18th century?

How to use this with your students: Distribute this example of Banneker's handwriting to your students and ask them to figure out what he did and why his method "works." Banneker used his journals to record astronomical observations, as his diary, and as his math notebook. Is the work in your students' math notebooks as clear as Banneker's?

References:

Adams, Daniel. *Scholars Arithmetic or Federal Accountant,* 1802, Leominster, MA

Bedini, Silvio A. *The Life of Benjamin Banneker: The First African American Man of Science,* 2nd ed. Baltimore, MD. Maryland Historical Society, 1999

Benjamin Banneker Association, http://www.mth.msu.edu/banneker/about.html

Eglash, Ron, "The African Heritage of Benjamin Banneker", in *Social Studies of Science,* Volume 27, Issue 2 (April 1997)

Fasanelli, Florence D, “Benjamin Banneker's Life and Mathematics: Web of Truth? Legends as Facts; Man vs. Legend,” a talk given on January 8, 2004, at the MAA/AMS meeting in Phoenix, AZ.

Lumpkin, Beatrice, “From Egypt to Benjamin Banneker: African Origins of False Position Solutions”, in *Vita Mathematica, Historical Research and Integration with Teaching* MAA Notes, Vol 40 (1996).

Lumpkin, Beatrice, “Mathematical Puzzles and Exercises from Banneker’s Manuscript Journal” an unpublished manuscript. Dr. Lumpkin taught at Malcolm X College in Chicago.

Mahoney, John F., “Benjamin Banneker’s Mathematical Puzzles” in NCTM’s Mathematics Teacher, Vol 96, #2, February, 2003.

Mahoney, John F., “Benjamin Banneker and Single Position” in NCTM’s Mathematics Teaching in the Middle School, Vol 10, #7, March, 2004

Mahoney, John F., “Benjamin Banneker and the Law of Sines” an article in the review process for NCTM’s Mathematics Teacher

Tyson, Martha E. *Banneker, The African-American Astronomer*, Philadelphia Friends Book Association, 1884. (Copy at the Maryland Historical Society)

How to use Benjamin Banneker's mathematical puzzles in your classroom

Banneker's puzzles can be solved by middle and high school students. Banneker didn't use symbolic algebra to solve these problems and these problems illustrate that there are some problems which are most easily solved without algebra! The mathematics department at my high school sponsored a contest centered on these puzzles. Each grade was assigned a problem to solve and there was one problem open to all students. The students had a number of weeks to solve the problems and Savings Bonds were awarded to randomly chosen correct entries. The contest was quite popular and brought added attention to Banneker's mathematics. We assigned the problems to the four grades as follows:

9^{th} Grade
10^{th} Grade
11^{th} Grade
12^{th} Grade
All Grades

About the author and Benjamin Banneker Academic High School

Benjamin Banneker Academic High School (www.benjaminbanneker.org) is a District of Columbia public school located on Euclid Street – a fitting location for perhaps the only high school in the country named for a mathematician. About 95% of the 390 students are African American. In June, 2002 Banneker HS was named #49 in Newsweek's list of America's Best High Schools. I introduced AP Statistics to the school in 2002 -2003 and taught 59 students that year in the course. Each of the students took the AP exam and I believe that we may have had the highest percentage of seniors taking AP Statistics in the country. This year I'm also teaching both AB level and BC level AP Calculus. I'm an AP Consultant and a long time participant in the AP Calculus Reading. Currently, I'm one of the exam leaders. I also chair the editorial panel of NCTM's ON-Math: www.nctm.org/onmath At Banneker High school I'm one of the mentors for our robotics team and as such I'm helping students design gear based drive trains – just as Banneker himself did over 200 years ago. Many engineers help us, including those from Howard University located across Georgia Avenue from us. I am one of the coaches of the school's award winning It's Academic team. When I decided to teach at Banneker three years ago, after a long career in private schools, I explored Banneker's mathematics and this paper is the result of that work.

Benjamin Banneker (November 9, 1731 â€“ October 19, 1806) was a free African-American almanac author, surveyor, landowner and farmer who had knowledge of mathematics and natural history. Born in Baltimore County, Maryland, to a free African-American woman and a former slave, Banneker had little or no formal education and was largely self-taught. He became known for assisting Major Andrew Ellicott in a survey that established the original borders of the District of Columbia, the federal capital district Banneker sent a manuscript copy in his own handwriting to Thomas Jefferson, then secretary of state and afterwards President of the United States. In addressing him he said: Those of my complexion have long been considered rather brutish than humanâ€”scarcely capable of mental endowments.Â Bannekerâ€™s abilities have often been brought forward as an argument against the enslavement of his race, and ever since he has been quoted as a proof of the mental capacity of Africans. Benjamin Banneker was an African American intellectual who constructed a wooden clock; predicted the solar eclipse of 1789; was an almanac author, a rare achievement for his time; and possessed advanced knowledge of astronomy and mathematics. Here are 10 interesting facts about the life and accomplishments of this famous American icon. #1 He was a free-born African American.Â #3 He primarily worked as a farmer his entire life. The Banneker family owned a 100-acre tobacco farm in the Patapsco River valley in rural Baltimore County. It was here that Benjamin was raised and spent nearly his entire life.Â Benjamin Banneker included his ephemeris in his series of almanacs which were titled Pennsylvania, Delaware, Maryland, and Virginia Almanac and Ephemeris. Banneker wrote a now-famous letter to Thomas Jefferson on August 19, 1791, arguing eloquently that " however variable we may be in Society or religion, however diversifyed in Situation or colour, we are all of the Same Family, and Stand in the Same relation to him [God]."1. In his reply to Banneker on August 30, Jefferson wrote, No body wishes more than I do to see such proofs as you exhibit, that nature has given to our black brethren, talents equal to those of the other colours of men, and that the appearance of a want of them is owing merely to the degraded condition of their existence b Benjamin Banneker is born in Baltimore County, Maryland to Robert, an ex-slave and Mary, the daughter of an Englishwoman and an African ex-slave. Benjamin is one of a couple hundred free blacks among a population of nearly 4000 slaves and 13,000 whites. 1753. Banneker constructs a striking clock.Â The brothers encourage Banneker to learn astronomy and mathematics. They loan him books and instruments for observing the stars. Banneker is able to teach himself astronomy and advanced mathematics. 1775. Banneker inherits the family farm.Â Banneker publishes the first of his almanacs. Banneker's almanacs are based on his calculated ephemeris. The almanacs also include literature, commentaries, and fillers that have a humanitarian and political nature.

Banneker wrote a now-famous letter to Thomas Jefferson on August 19, 1791, arguing eloquently that "...however variable we may be in Society or religion, however diversifyed in Situation or colour, we are all of the Same Family, and Stand in the Same relation to him [God]."1. In his reply to Banneker on August 30, Jefferson wrote, No body wishes more than I do to see such proofs as you exhibit, the nature has given to our black bretheren, talents equal to those of the other colours of men, and that the appearance of a want of them is owing merely to the degraded condition of their existence bot in Africa and America...I have taken the liberty of sending. Banneker himself surely suffered discriminated in his life, having his great achievements dismissed and belittled by people who couldnâ€™t probably have understood them but still considered themselves superior because of the color of their skin. All records indicate, however, that Banneker never let them get him down. â€œHis equilibrium was seldom disturbed by the petty jealousies and inequalities of temper of the ignorant people,â€A Sketch of the Life of Benjamin Banneker notes, â€œwith whom his situation obliged him frequently to come in contact.â€ Benjamin Ellicott, who knew him personally, remembered him similarly in a letter Benjamin Banneker, originally Banna Ka, or Bannakay (November 9, 1731 â€“ October 9, 1806) was a free African American mathematician, astronomer, clockmaker, and publisher. He was America's first African American scientist and a champion of civil rights and world peace. Benjamin Banneker was born near Ellicot City, Maryland, on November 9, 1731. He was the first of three children to Robert, a freed slave from West Africa, and Mary Banneky, of English-African descent. Mary was the second of four... In his journals, Banneker wrote and collected mathematical puzzles written in verse. These journals served as his notebooks for astronomical observations, his diary, and his math notebook. Unfortunately only one of his journals survived a fire on the day of his funeral. The mathematics in this journal consisted of six puzzles and two pages of mathematical writing.Â To my knowledge, much of Banneker's mathematics in his own handwriting has never been reproduced. I located a microfilmed copy of his journal at the Maryland Historical Society in Baltimore. The quality of the reproduction was poor, but I was helped by Mr. Omar Rumi of Kuala Lumpur, Malaysia, and my son, Quinn, a student at MIT. Benjamin Banneker (November 9, 1731 â€“ October 9, 1806) was a free African American astronomer, mathematician, surveyor, almanacauthor and farmer. Family history and early life It is difficult to verify much of Benjamin Banneker's family history.Â Bannekerâ€™s activities on the survey team resembled those used in celestial navigation during his lifetime. His duties consisted primarily of making astronomical observations at Jones Point in Alexandria, Virginia, to ascertain the location of the starting point for the survey and of maintaining a clock that he used when relating points on the surface of the Earth to the positions of stars at specific times.[12] Because of illness and the difficulties in helping to survey the area.