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2–Domination in Fuzzy Graphs

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Abstract. In this paper, 2–dominating set and 2–domination number of a fuzzy graph are introduced. The 2–domination number γ_2(G), of the fuzzy graph G is the minimum cardinality taken over all 2–dominating sets of G. We also prove some results on 2-dominating set. The exact values of γ_2(G) for some standard fuzzy graphs are found.

Keywords: Strong neighbours, 2 – dominating set, 2 – domination number

AMS Mathematics Subject Classification (2010): 03E72, 05C72

1. Introduction

The study of dominating sets in graphs was started by Ore and Berge [1,9]. The domination number and the independent domination number were introduced by Cockayne and Hedetniemi [3]. The n-domination in graphs was introduced by Fink and Jacobson [4] in the year 1985. The concept of fuzzy relation was introduced by Zadeh [12] in his classical paper in 1965. Rosenfeld [10] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness.

Somasundram and Somasundram [11] discussed domination in fuzzy graphs. They defined domination using effective edges in fuzzy graphs. NagoorGani and Chandrasekaran [5] discussed domination in fuzzy graph using strong arcs. NagoorGani and Vadivel [7,8] discussed domination, independent domination and irredundance in fuzzy graphs using strong arcs. NagoorGani and Prasanna Devi [6] discussed edge domination and edge independence in fuzzy graphs.

2. Preliminaries

A *fuzzy graph* G $=<\sigma,\mu>$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V\times V \to [0,1]$, where for all x,y ∈ V, we have $\mu(x,y) \le \sigma(x)\wedge\sigma(y)$. A fuzzy graph H $=<\tau,\rho>$ is called a *fuzzy subgraph* of G if $\tau(v_i) \le \sigma(v_i)$ for all v_i ∈ V and $\rho(v_i,v_j) \le \mu(v_i,v_j)$ for all v_i,v_j ∈V. The *underlying crisp graph* of a fuzzy graph G $=<\sigma,\mu>$ is denoted by G^* =<σ*, μ*>, where σ*= {v_i ∈V / σ(v_i)>0} and μ* ={(v_i,v_j) ∈ V×V / μ(v_i,v_j) >0}. An edge in G is called an *isolated edge* if it is not adjacent to any edge in G. A node in G is called an *isolated node* if it is not adjacent to any node in G. A *path* with n vertices in a fuzzy graph is denoted as P_n . A fuzzy graph G $=<\sigma,\mu>$ is a *complete fuzzy graph* if

$\mu(v_i, v_j) = \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in \sigma^*$. An arc (x, y) in a fuzzy graph G =<σ,μ> is said to be *strong* if $\mu^{\infty}(x, y) = \mu(x, y)$ then x, y are called *strong neighbours*. The *strong neighbourhood* of the node u is defined as $N_S(u) = \{v \in V: (u, v)$ is a strong arc$\}$. A subset D of V is called a *dominating set* of a fuzzy graph G if for every $v \in$ V− D, there exist $u \in$ D such that u dominates v. The *domination number*, γ(G), of a fuzzy graph G, is the smallest number of nodes in any dominating set of G.

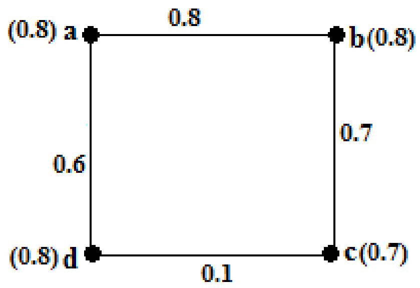
The *2-dominating set* D of a graph is defined as if for every node $v \in$ V−D there exist atleast two neighbours in D. In this paper we discuss about 2-dominating set and 2-domination number of a fuzzy graph.

3. 2-dominating set

In this section, we define 2-dominating set and 2-domination number of a fuzzy graph with suitable examples. We also derive some results on the 2-domination number of the fuzzy graphs.

Definition 3.1. A subset D of V is called a ***2-dominating set*** of G if for every node $v \in$ V−D there exist atleast two strong neighbours in D.

Example 3.2.



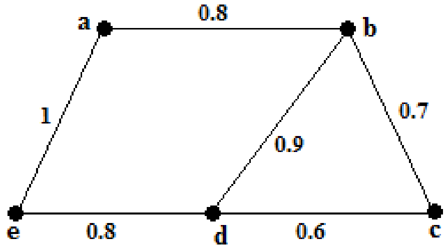
(G)

Figure 3.1:

$\{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$ and $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$are 2-dominating sets of the fuzzy graph G.

Definition 3.3. The ***2-domination number*** of a fuzzy graph G denoted by γ_2(G), is the minimum cardinality of a 2-dominating set of G.

Example 3.4.



(G)

Figure 3.2:

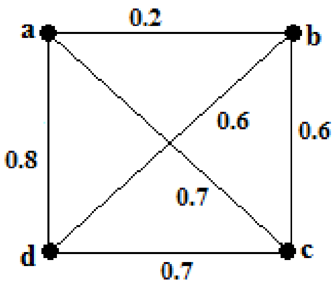
$\{\boldsymbol{b},\boldsymbol{c},\boldsymbol{e}\},\{\boldsymbol{a},\boldsymbol{c},\boldsymbol{d}\},\{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{e}\},\{\boldsymbol{b},\boldsymbol{c},\boldsymbol{d},\boldsymbol{e}\},\{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d}\},\{\boldsymbol{a},\boldsymbol{c},\boldsymbol{d},\boldsymbol{e}\}$ and $\{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d},\boldsymbol{e}\}$ are 2-dominating set of the fuzzy graph G.

$\Longrightarrow\ \gamma_2(\mathrm{G}) = 3.$

Definition 3.5. A 2-dominating set D of a fuzzy graph G such that $|\mathrm{D}| = \gamma_2(\mathrm{G})$ is called a ***minimum2-dominating set*** of G.

A 2-dominating set D is called a ***minimal 2-dominatingset*** if no proper subset of D is a 2-dominating set of G.

Example 3.6.



(G)

Figure 3.3:

Here $\{\boldsymbol{c},\boldsymbol{d}\},\{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\},\{\boldsymbol{a},\boldsymbol{b},\boldsymbol{d}\}$and $\{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d}\}$ are all 2-dominating set of the fuzzy graph G.

The sets $\{\boldsymbol{c},\boldsymbol{d}\},\{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\}\ and\ \{\boldsymbol{a},\boldsymbol{b},\boldsymbol{d}\}$ are minimal 2-dominating sets of G.

The set $\{\boldsymbol{c},\boldsymbol{d}\}$ is the minimum 2-dominating set of G.

Theorem 3.7. If $|N_S(v)| \leq 1$, then v belongs to every 2-dominating set of the fuzzy graph G.

Proof: Let G be a fuzzy graph and v ∈V has atmost one strong neighbour in G. Let D be a 2-dominating set of G.

Case (i):

Suppose v has no strong neighbours in G. i.e.,$N_S(v) = \emptyset$.

Then v is not dominated by any node in D, since v is an isolated node in G. Thus v should be dominated by itself.

Hence v belongs to every 2-dominating set of G.

Case (ii):

Suppose v has only one strong neighbour in G.

Suppose $v \notin$ D.

Then v has atmost one strong neighbour in D. But D is a 2-dominating set of G i.e., every node $v \in$ V−D has atleast two strong neighbours in D. Since $v \notin$ D and has atmost one strong neighbour in D then D is not a 2-dominating set of G, which is a contradiction to our assumption.

Therefore, $v \in$ D, for every 2-dominating set D of G.

Theorem 3.8. Every 2-dominating set of a fuzzy graph G is a dominating set of G.

Proof: Let D be a 2-dominating set of the fuzzy graph G. Then every node in V− D has atleast two strong neighbours in D i.e., for every node $v \in$ V−D, there exist minimum two nodes in D and both dominates v.

Every node in V−D is dominated by atleast two nodes in D. Thus D is a dominating set of G.

Corollary 3.9. If G is a fuzzy graph then γ_2(G) ≥ γ(G).

Proof: By the above theorem, every 2-dominating set of a fuzzy graph G is a dominating set of G. Thus every minimum 2-dominating set of G is also a dominating set of G.

Therefore, γ_2(G) ≥ γ(G).

Theorem 3.10. Every connected fuzzy graph G has a minimum 2-dominating set D then V – D need not be a 2-dominating set of G.

Proof: Let D be a 2-dominating set of G and let $v \in$ V.

Suppose $|N_S(v)| = 1$, then v belongs to every 2-dominating set of G.

Thus v belongs to every minimum 2-dominating set of G. Then V − D has either no strong neighbour of v or only one strong neighbour of v. Thus V – D does not has two strong neighbours for v.

⇒ V – D is not a 2-dominating set of G.

Suppose every node in D has atleast two strong neighbours in V − D.

Then in this case, every node in D has atleast two strong neighbours in V− D. Thus V− D is a 2-dominating set of G.

Hence V – D need not be a 2-dominating set of G.

4. 2-domination number for standard fuzzy graphs

In this section we discuss about the 2-dominating set and 2-domination number of some standard fuzzy graphs.

Theorem 4.1. If $e = (u, v)$ is an isolated edge in a fuzzy graph G then both u and v are in every 2-dominating set of G.

Proof: Let $e = (u, v)$ is an isolated edge in a fuzzy graph G. then e is a strong arc in G i.e., u and v are strong neighbours in G.

⇒ $N_S(u) = \{v\}$and$N_S(v) = \{u\}$

⇒ $|N_S(u)| = 1$and$|N_S(v)| = 1$

Since$|N_S(v)| = 1$, then u belongs to every 2-dominating set of G. Simillarly v belongs to every 2-dominating set of G.

Thus both u and v are in every 2-dominating set of G.

Corollary 4.2. If G is a fuzzy graph and G*= nK_2 then γ_2(G) $= 2n$.

Proof: Let G be fuzzy graph and G*= nK_2.

K_2 is a complete fuzzy graph with two nodes i.e., a strong arc with its nodes.

By above theorem, both the vertices of K_2 is in the 2-dominating set of G.

Thus the fuzzy graph G has all the $2n$ nodes in the 2-dominating set of G and also it will be in the minimum 2-dominating set of G.

Therefore, γ_2(G) $= 2n$.

Theorem 4.3. If K_n is a complete fuzzy graph, $n \geq 2$, then $\gamma_2(K_n) = 2$.

Proof: K_n is a complete fuzzy graph with n nodes. Here every node in K_n is a strong neighbour to all other nodes in it.

Thus any two nodes in K_n will form a 2-dominating set of K_n and it will be a minimum 2-dominating set of K_n.

Therefore $\gamma_2(K_n) = 2$.

Theorem 4.4. If G is a fuzzy graph and G* is a cycle with n nodes then the 2-domination number of G,

$$\gamma_2(\mathrm{G}) = \begin{cases} \left\lfloor \frac{n+1}{2} \right\rfloor, & \textit{if G has more than one weakest arc.} \\ \left\lfloor \frac{n}{2} \right\rfloor + 1, & \textit{if G has only one weakest arc.} \end{cases}$$

Proof: Let G be a fuzzy graph and G* be a cycle with n nodes. Let $v_1, v_2, \ldots \ldots, v_n$ be the n nodes of G.

Case (i): If G has more than one weakest arc then all the arcs of G are of strong arcs. Then $\left\lfloor \frac{n+1}{2} \right\rfloor$ nodes in G will form a 2-dominating set of G.

Therefore $\gamma_2(G) = \left\lfloor \frac{n+1}{2} \right\rfloor$.

Case (ii): If G has only one weakest arc then G has $n-1$ strong arcs and only one non strong arcs.

If $e = (v_1, v_2)$ is the only one non strong arc in G then $|N_S(v_1)| = 1$ and $|N_S(v_2)| = 1$. Thus both v_1 and v_2 will be in every 2-dominating set of G. Therefore $\left\lfloor \frac{n}{2} \right\rfloor + 1$ nodes in G will form a 2-dominating set of G.

Therefore $\gamma_2(G) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Corollary 4.5. If P_n is a fuzzy path with n nodes then $\gamma_2(G) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Theorem 4.6. Let G be a fuzzy graph and G* is a star with $n+1$ nodes, $n \geq 2$, then $\gamma_2(G) = n$.

Proof: Let G be a fuzzy graph and G* is a star with $n+1$ nodes.

The nodes except the centre node will have only one strong neighbour and it will be in every 2-dominating set of G. And the centre will have all other n nodes as its strong neighbours. The nodes except the centre node will form a 2-dominating set and it will be the minimum. Therefore $\gamma_2(G) = n$.

5. Conclusion

We defined 2-dominating set and 2-domination number of a fuzzy graph. For some standard fuzzy graphs, we have given the exact value of the 2-domination number. Further works are to find the relation between 2- domination number with edge domination number of fuzzy graphs.

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The study of dominating sets in graphs was begun by Orge and Berge. V. R. Kulli [10] wrote on theory of domination in graphs. A. Somasundaram, S. Somasundaram [9] presented the concepts of Domination in fuzzy graphs. Here we introduced the concept of Excellent domination in fuzzy graphs and their related concepts. 2. Preliminaries. Definition 2.1.Â In this case the resulting graph is fuzzy excellent graph H. Clearly G is an induced subgraph of H, Î³f (H) = Î³f (G) + 1. For every Î³f -set D of G, D âˆª {t} and D âˆª {t1} are Î³f (H)-sets of G. Therefore H is Î³f -fuzzy excellent. Case 2: Assume that |T | 2 and T = {t1, t2, ..., tn}. Now we assume that there is a non-empty subset T âˆ— of T such that T âˆ— is extreme optimal fuzzy bad set. Keywords: Direct product; Fuzzy graph; Domination number; Total domination number. Mathematics Subject Classication: 05C72; 03E72; 03F55. 1 Introduction with preliminaries.Â The theory of domination and total domination in product graphs is a rich topic of investigation both from the point of view of theory and application. Com-puter science is a branch in which the use of product graphs have become indispensable, specically, in problems such as load balancing for massively parallel computer architec-tures. In this paper we have dened the concept of direct product of two fuzzy graphs and proved some important results related to the theory of domination in this new set up. The concepts have also been illustrated with precise examples. In graph theory, a dominating set for a graph G = (V, E) is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number Î³(G) is the number of vertices in a smallest dominating set for G. The dominating set problem concerns testing whether Î³(G) â‰¤ K for a given graph G and input K; it is a classical NP-complete decision problem in computational complexity theory. Therefore it is believed that there may be no efficient algorithm that finds a smallest...