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A Hybrid Power Series – Artificial Bee Colony to Solve Magnetohydrodynamic Viscous Flow Over a Nonlinear Permeable Shrinking Sheet

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Abstract – *A hybrid Power series and Artificial Bee Colony Algorithm (PS-ABC) is applied to solve a system of nonlinear differential equations arising from the similarity solution of magnetohydrodynamic viscous flow over a nonlinear permeable shrinking sheet. The flow is subjected to variable magnetohydrodynamic effects. A similarity solution is obtained for the boundary layer governing equations. Then, a trial solution of the governing differential equation is defined as sum of two polynomial parts. The first part satisfies the boundary conditions and contains an adjustable parameter and the second part which is constructed so as not to affect the boundary conditions and involves adjustable parameters (Coefficients of polynomial of the second part). The artificial bee colony algorithm is applied to find adjustable parameters of trial solution (in first and second part). The problem successfully is solved using proposed method. The obtained solution is compared with numerical solution as well as exact solution in a case which closed form exact solution exists. The main objective is to propose a power series solution which do not require small parameters and avoid linearization and physically unrealistic assumptions. In addition, the results of presented method represent a remarkable accuracy in compare with exact solution and numerical results. Present method can be easily extended to solve a wide range of boundary layer problems.*

***Keywords**: MHD, Artificial Bee Colony, Similarity Solution, Power Series, Variable Magnetic*

Nomenclature

a	Vector of adjustable parameters
B	Applied magnetic field
D	Domain of solution
E	Error
f	Similarity function for stream function
K	Mass transfer parameter
M	Magnetic parameter
m	Constant value
N	Power series
u	Velocity component in x-direction
v	Velocity component in y-direction
ν	Kinematic viscosity
V_0	Porosity of the sheet
x,y	Cartesian coordinate system
Y_T	Trial solution
β	Non-dimensional parameter
η	Similarity variable
λ	Dummy parameter
ρ	Density of fluid
σ	The electrical conductivity of the fluid

I. Introduction

Most problems in fluid mechanic and physics are inherently of nonlinearity. One of these problems is the phenomena of velocities on the boundary towards a fixed point which the flow is subjected to magnetohydrodynamic effects. The phenomena of velocities on the boundary towards a fixed point can be described as shrinking phenomena [1]-[4]. The magnetohydrodynamic flows are important in various areas of technology and engineering such as MHD power generation, MHD pumps and MHD flow meters and extrusion processes in plastic and metal industries [2], [3].

Effects of magnetic fields on the hydrodynamic flow in various configurations are studied and discussed by previous researchers [1]. The problem of two dimensional electrically conducting viscous fluid pas a porous nonlinear porous shrinking sheet with variable magnetic field effect was studied by Nadeem and Hussain [1]. The governing partial differential equations can be transformed to a third order nonlinear ordinary differential equation using similarity variables [1]. Many different methods have been developed for solving differential equations. However, the solution of nonlinear differential equations is still challenging [5], [6]. Recently new artificial methods have attracted the

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attention of researchers to solve engineering problems [6]-[12].

Lee and Kang [13] used parallel processor computers in order to solve a first order differential equation using Hopfield neural network models. Meade and Fernandez [14] used feed forward neural networks architecture and B_1 splines to solve linear and nonlinear ordinary differential equations. Lagaris et al. [15] represented a new method to solve First order linear ordinary and partial differential equations using artificial neural networks. Malek and Beidokhti [16] used a hybrid artificial neural network- Nelder-Mead method to solve high order linear differential equations. A hybrid artificial neural network- swarm intelligence method was used by Khan et al. [5] to solve Wessinger's equation.

In the present work, a hybrid polynomial Power Series and Artificial Bee Colony (PS- ABC) is used to solve the nonlinear differential equation arising from studies of conducting viscous fluid pas a nonlinear permeable shrinking sheet with variable magnetic effects. The effects of various parameters of wall mass transfer and magnetic strength on the velocity profiles and skins fractions are considered. The presented method reduces the computational difficulties of the other methods. The obtained results are compared with the results of HAM solution in the work of Nadeem and Hussain [1] as well as a numerical method.

II. Mathematical Model

Consider a two-dimensional magnetohydrodynamic (MHD) viscous flow over a nonlinear permeable shrinking sheet at y=0, which is incompressible and steady state. The fluid is electrically conducting under the influence of an applied magnetic field $B(x)$ normal to the shrinking sheet. By neglecting the induced magnetic field and the polarization effects and external electric field, the governing equations of continuity and momentum in the boundary layer can be written as follows [1]:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial x}=\upsilon\frac{\partial^2 u}{\partial x^2}-\sigma\frac{A_0^{\,2}(x)}{\rho}u \tag{2}$$

subject to the following boundary conditions:

$$u(x,0)=-cx^m;\ \upsilon(x,0)=-V_0 t^{\frac{m-1}{2}};\quad u(x,y)\to 0 \quad \text{as } y\to\infty \tag{3}$$

where u and v are the velocity components, parallel to the surface (x) and perpendicular to the surface (y), respectively. υ is the kinematics viscosity, σ is the electrical conductivity of the fluid, ρ is the density and V_0 is the porosity of the sheet.

The magnetic field can be defined in the following form:

$$A(x)=A_0 x^{\frac{m-1}{2}} \tag{4}$$

Now, by introducing the following similarity variables and non-dimensional variables:

$$\eta=\left(\frac{d(m+1)}{2\upsilon}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}},\ u=dx^m f'(\eta) \tag{5}$$

$$\upsilon=-\left(\frac{\upsilon d(m+1)}{2\upsilon}\right)^{\frac{1}{2}} x^{\frac{m-1}{2}}\left(f(\eta)+\gamma\frac{m-1}{m+1}f'(\eta)\right) \tag{6}$$

and applying them to (1), (2) and (3) the continuity equation is automatically satisfied, and the momentum equation is transformed to:

$$f'''+ff''-\beta f'^2-M f'=0 \tag{7}$$

subject to:

$$f(0)=K\quad,\ f'(0)=-1\ ,\ \ f'(\infty)=0 \tag{8}$$

where K is the mass transfer parameter due to permeability of the surface, M is the magnetic parameter, and β is a non-dimensional parameter.

These parameter are introduced as follows:

$$\beta=\frac{2m}{m+1},M=\frac{2\sigma A^2_{\ 0}}{\rho d(1+m)},\ K=\frac{V_0}{\left(\frac{\upsilon d(m+1)}{2\upsilon}\right)^{\frac{1}{2}}} \tag{9}$$

It is worth mentioning that in the case of β=1 the exact solution of (7) was obtained by Fang [17] as:

$$f(\eta)=K-\frac{1}{\lambda}+\frac{1}{\lambda}e^{\lambda\eta} \tag{10}$$

where:

$$\lambda=\left(K\pm\sqrt{K^2-\left(4-4M^2\right)}\right)\Big/2$$

III. Artificial Bee Colony Optimization Algorithm

The Artificial Bee Colony (ABC) algorithm, proposed by Karaboga in 2005 for real parameter optimization, is a gradient free optimization algorithm, which inspired from intelligent foraging behavior of a bee colony [18]. In a natural bee colony, some tasks are performed by specialized individuals. In the ABC algorithm, the colony of artificial bees contains three groups of bees: employed bees, onlookers and scouts [19].Onlooker is a bee waiting for making decision to choose a food source. A bee which is going to the food source visited by itself previously is named an employed bee. A scout bee is a bee which carrying out random search around the hive. In the ABC algorithm, first half of the colony consists of employed artificial bees and the second half constitutes the onlookers. The number of employed bees is equal to the number of food sources around the hive. The employed bee whose food source is exhausted by the employed and onlooker bees becomes a scout. The main steps of the algorithm can be summarized as [19]:

• Initializing.
• REPEAT.
(a) Place the employed bees on the food sources in the memory;
(b) Place the onlooker bees on the food sources in the memory;
(c) Send the scouts to the search area for discovering new food sources.
• UNTIL (requirements are met).

These specialized bees try to maximize the nectar amount stored in the hive using efficient division of labor and self-organization. The basic idea and details of ABC algorithm are explained in [18]-[20]. In the ABC algorithm proposed by Karaboga, the position of a food source represents a possible solution to the optimization problem, and the nectar amount of a food source corresponds to the profitability (fitness) of the associated solution. In this study, ABC algorithm is coded with MATLAB 2007.

IV. Problem Formulation

Consider governing equation of MHD viscous flow which it is expressed by (7) subject to (8). In order to solve (7), assume a discretization of the domain D with m arbitrary points. Here, the problem can be written as the following set of equations [15], [16], [21]-[24]:

$$\begin{aligned}&\frac{d^3u(x_i)}{dx^3}+u(x_i)\frac{d^2u(x_i)}{dx^2}+\\&-\beta\left(\frac{du(x_i)}{dx}\right)^2-M\frac{du(x_i)}{dx}=0\\&x_i\in D,\quad i=1,2,...,m\end{aligned}\tag{11}$$

subject to given boundary conditions (i.e. (8)).

Let's assume $y_T(x,\vec{a})$ as an approximate solution to (7) where, $\vec{a}$ is a vector which contains adjustable parameters. These parameters (i.e. adjustable parameters) should be determined by minimizing the following sum of squared errors, subject to given conditions in (8):

$$\begin{aligned}&E(\vec{a})=\\&=\sum_{i=1}^{m}\begin{pmatrix}\dfrac{d^3y_T(x_i,\vec{a})}{dx^3}+y_T(x_i,\vec{a})\dfrac{d^2y_T(x_i,\vec{a})}{dx^2}+\\-\beta\left(\dfrac{dy_T(x_i,\vec{a})}{dx}\right)^2-M\dfrac{dy_T(x_i,\vec{a})}{dx}\end{pmatrix}^2\\&x_i\in[0\ \ \infty]\end{aligned}\tag{12}$$

In order to transform (11) to an unconstrained problem $y_T(x,\vec{a})$ can be written in the following form:

$$Y_T(x,\vec{a})=\begin{pmatrix}a_1x^3+\dfrac{1-3a_1b^2}{2b}x^2-x+\\+K+x^2(x-b)^2N(x,\vec{a})\end{pmatrix}\tag{13}$$

where a_1 is an adjustable parameters and b is a large number. $N(x,\vec{a})$ is an infinite power series $\left(N(x,\vec{a})=\sum_{i=0}^{n}a_{i+2}x^i\right)$ which involves adjustable parameters ($a_2 \ldots a_n$). Now equation (12) is in the form of a finite power series with adjustable coefficients which exactly satisfy given boundary conditions of (8). Now an optimization technique like ABC can be applied in order to determine optimal adjustable parameters of $y_T(x,\vec{a})$ (i.e. $\vec{a}$) to minimize $E(\vec{a})$ in (12).

V. Results and Discussion

By taking step size of 0.1 in the domain of solution *D*, 61 collocation points are selected for all cases ($x_i \in \{0, 0.1, 0.2 \ldots b\}$) in the following text. For the best given results, Following combination of user-specified parameters of ABC method are used for this problem. The number of colony size (employed bees + onlooker bees) is 150 and the food source limit which could not be improved through trials will be abandoned by its employed bee after 100 try. Finally the maximum foraging try is 450. In order to demonstrate the accuracy of the presented method, comparison is made between the presented solution and numerical results as well as exact solution when β=1. Numerical results are obtained using a combination of trapezoid as base scheme and Richardson extrapolation as enhancement scheme [25]. It is found that there are more than one solution branches under certain circumstances. However, in the present study only one solution branch has been considered. It is found that this method with n=6, asymptotical large number of b equal to six, provides results with very good agreement with numerical results. Therefore solution with eight variables ($a_1..a_8$) and b=6.0 are used in the following text. For instance an obtained values of variable a when M=4, β=1 and K=0.5 are shown in Table I.

TABLE I
OPTIMIZED VALUES OF VECTOR A WHEN M=4, β=1 AND K=0.5

Variables	Value	Variables	Value
a_1	-0.0469	a_5	-0.0025
a_2	0.0371	a_6	0.0005
a_3	-0.0202	a_7	-0.0001
a_4	0.0082	a_8	0

Variation of $f\ '(\eta)$ (velocity profiles) for different values of parameter K (mass transfer) when M=1 and β=1 has been shown in Fig. 1 and compared with exact solution. It is observed that the boundary layer thickness decreases with increase of parameter K. Increase of mass injection (increase of positive values of K) decreases the magnitude of velocity profiles. Variation of velocity profiles for different values of magnetic effect when β=0.6 and K=4 has been shown in Fig. 2.

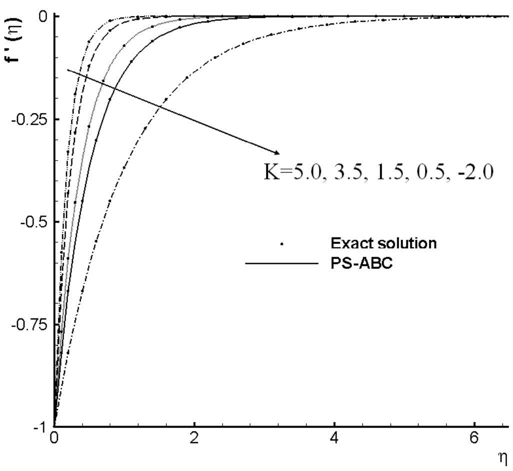

Fig. 1. Variation of velocity profiles for different values of mass transfer parameter when M=1, β=1

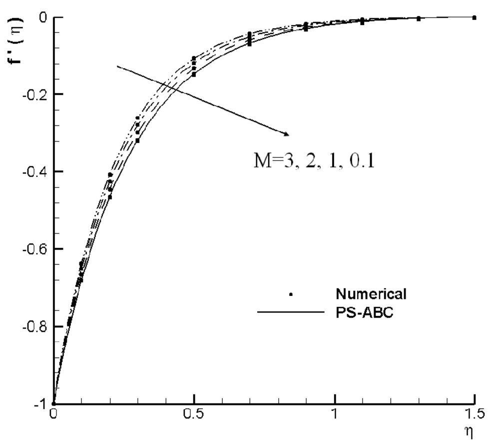

Fig. 2. Variation of velocity profiles for different values of magnetic parameter when β=0.6 and K= 4.0

This figure depicts that increase of magnetic parameter decreases the momentum boundary layer thickness and the magnitude of $f\ '(\eta)$ decreases with increase of parameter M. The effect of non dimensional parameter of β on the velocity profiles is shown in Fig. 3. This figure shows that increase of β increases the magnitude of velocity profiles and boundary layer thickness.

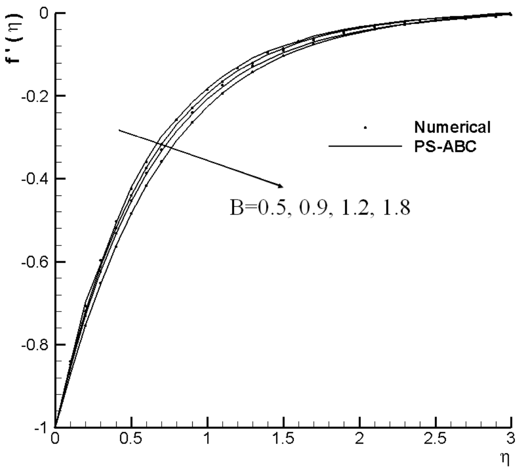

Fig. 3. Velocity profiles for different values of parameter β when M=2, K=1

VI. Conclusion

Nonlinear differential equation arising from similarity solution of MHD viscous flow due to a nonlinear permeable shrinking sheet and variable magnetic effect was solved using a hybrid polynomial power series and artificial bee colony optimization algorithm. ABC was applied in order to find the adjustable parameters of trial function regarding to minimize a fitness function including these parameters (i.e. adjustable parameters). These parameters were determined so that the trial function has to satisfy the boundary conditions. The obtained solution in comparison with the numerical ones shows very good agreement.

The proposed approach is quite general. The results show that the momentum boundary layer thickness decreases by increasing the values the magnetic field or mass injection, but it increases with increase of β. The future work is focused on comparing the effect of changing optimization technique on minimizing the function of $E(a)$.

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This work deals with the unsteady micropolar fluid over a permeable curved stretching and shrinking surface. Using similarity transformations, the governing boundary layer equations are transformed into the nonlinear ordinary (similarity) differential equations. The transformed equations are then solved numerically using the shooting method. The effects of the governing parameters on the skin friction and couple stress are illustrated graphically.Â and unsteady flows of a viscous incompressible fluid determined by a linearly stretching surface through a quiescent fluid. Such flow situations are encountered in a number of manufacturing practices, such as the cooling of metallic plate in a cooling bath, polymer sheet extrusion, and heat-treated materials that are on a conveyor belt. 3. Conclusion Viscous flow over a continuously shrinking sheet has been solved using a second order slip flow model proposed by Wu [45]. A closed form analytical solution which represents an exact solution of the governing Navierâ€“Stokes equations has been derived. A critical mass transfer parameter scritical has been identified for the solution to exist. In the present study, a combination of power series and artificial bee colony optimization algorithm is applied to obtain a power series solution for the nonlinear ordinary differential equations of MWCNT cantilevers. A remarkable accuracy for the presented method is achieved when the obtained results are compared with numerical results.Â A. Noghrehabadi, M. Ghalambaz, and M. Ghalambaz, â€œA hybrid power seriesâ€”artificial bee colony to solve magnetohydrodynamic viscous flow over a nonlinear permeable shrinking sheet,â€ International Review on Modelling and Simulations, vol. 4, no. 5, pp. 2696â€“2700, 2011. View at: Google Scholar. Magnetohydrodynamic flow of Cuâ€“Fe3O4/H2O hybrid nanofluid with effect of viscous dissipation: dual similarity solutions. Liaquat Ali Lund 1,2 , Zurni Omar 1Â MHD boundary layer flow and heat transfer of nanofluids over a nonlinear stretching sheet: a numerical study. J Magn Magn Mater. 2015;374:569â€“76. Noghrehabadi, A., Ghalambaz, M., Ghalambaz, M., A hybrid power series - Artificial bee colony to solve magnetohydrodynamic viscous flow over a nonlinear permeable shrinking sheet, (2011) International Review on Modelling and Simulations (IREMOS), 4 (5), pp. 2696-2700. Kermani, E.P., Jahanshaloo, L., Sidik, N.A.C., Analysis of mixed convection of alumina-water nanofluid flow over heated cavity using lattice Boltzmann method, (2013) International Review on Modelling and Simulations (IREMOS), 6 (4), pp. 1350-1354. M.H. Yazdi, S. Abdullah, I. Hashim, K. Sopian, Slip MHD Liquid Flow and Heat Transfer Over Non-Linear Permeable Stretching Surface With Chemical Reaction, International Journal of Heat and Mass Transfer, 54,15-16, 3214-3225, 2011.